The number sense: a comparison between normal and adapted curriculum pupils in terms of subitization ability, the distance and magnitude effects

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***Abstract*:** Number sense refers to the ability to easily understand, approximate and manipulate numerical quantities. In solving numerical problems, children use exact, respectively approximate calculation strategies. Unfortunately, some persons encounter difficulties in acquiring basic mathematical skills, and this condition represent a specific learning disability called developmental discalculia. The aim of the present study was to compare normal and adapted curriculum pupils in terms of subitization ability and efectiveness of number sense. The presence of magnitude and distance effects was tested in four distinct formats: 3D objects, 2D figures, verbal and visual Arabic numerical representation. The sample consisted of 111 normal and adapted curriculum pupils. Both number of errors and reaction times were registered. Adapted curriculum pupils registered more erroneous answers in the 2D, verbal, and arabic tasks and bigger reaction times in the 3D, tasks with Arabic or verbal numerals. Both normal and adapted curriculum pupils registered the best performances in the visual Arabic numerical representation task. Contrary to the expectations, the magnitude effect was not outlined in terms of reaction times, perhaps due to the calculation strategies which pupils adopted in different tasks.

***Keywords*:** Number sense, Specific learning disability with impairment in mathematics, Subitization, Distance effect, Magnitude effect

Introduction

Mathematics, defined as the „science of quantity, or of quantitative relations” (Kosc, 1970, pp. 12-13), build themselves, with a large support of the language, on a primary, perhaps biologically ability, called the number sense

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(Dehaene, 1997). The number sense is claimed to be an universal characteristic of human beings and, more than that, it seems to be shared across species. Numerical sense refers to the ability to easily understand, approximate and manipulate numerical quantities (David & Roșan, 2017). Some authors defined the term of number sense as the ability to picture and manipulate numerical magnitude on an internal numerical axis, with small values in the left side and the big ones in the right (Gelman & Gallistel, 1978; Stocket. al., 2010). It is an internal and informal representation of the numerical quantities (Murphy & Mazzocco, 2008), very useful for survival. Dehaene (2000, p. 988) considered thatarithmetic knowledge, namely the number sense, refers to „a highly specific and limited mode of representation and manipulation of numbers, in the form of abstract quantities laid down on an analogical numerical axis". Gersten and Chard (1999, p.19) explained the number sense as „fluidity and flexibility with numbers, the sense of what numbers mean, and an ability to perform mental mathematics and to look at the world and make comparisons". The number sense makes possible the understanding of the meaning of numbers, learning symbols for numbers (Dehaene, 2000), making number comparisons, subitizing, operating numbers, developing strategies and interpreting numerical information (Stocket. al., 2010). The existence of a spatial – oriented mental numerical axis is supportedby the SNARC effect. Spatial numerical association of response codes (SNARC) effect refers to observation that people make faster judgments about a number if the hand used to respond is congruous with the size of the number in question (left hand for small numbers, respectively right hand for =bigger ones). Four steps are identifiable in the development of the number sense:(1) representation of numerical magnitude / cardinality, (2) acquisition of linguistic number symbols, (3) acquisition of Arabic number symbols, and (4) expansion of a mental numerical axis (Von Aster & Shalev, 2007).

Exact number system vs. approximate number system

In solving problem situations, children use two different types of strategies: exact calculation strategies, respectively approximate calculation strategies (Rousselle & Noel, 2008). The approximate number system and the exact number system are fundamentally linked (Feigenson et al., 2013). Exact number system implies counting and calculation, while approximate number system refers to contextual and perceptual estimation (Feigenson et al., 2004; McCrink & Wynn, 2004, apud. Olkun et al., 2015). The presence of an approximate number system was proven not just in adults and children, but also in infants. 6 months old infants are able to successfully discriminate 8 vs. 16 and 16 vs. 32 dots, but fail with 8 vs. 12 and 16 vs. 24, when a habituation paradigm is used (Xu & Spelke, 2000). The approximate number system generates a noisy representation of approximate number, regardless the modality of presentation or the variations in continous properties (Feigenson et al., 2004).On the other hand, subitizing, defined as the rapid aprehension of the numerosity of sets usually smaller than four, despite the fact it was included in the exact number system with counting and calculation, represents the basic process of determining quantities exclusive in a primarily perceptive way (Dehaene, 2000). Subitization is also present in infants (Feigenson et al., 2004).

The triple code model

In conformity to the triple code model (Dehaene & Cohen, 1995), mathematical processing is realized through the implication of three codes or formats that people can use to represent the magnitudes. First of them is an analogical system of representation of quantities, and it conceives the numbers as a distribution of activation on a mental numerical axis, bilaterally localized in the inferior parietal region. The second code is a verbal one (phonological and graphical), representing the numbers in sequences of words syntactically organized. This representation is correlated with the perisylvian cortex, implicated in the verbal processing. The third code is a visual arabical one, with an ideographical character, in which numbers are represented as series of digits; it allows a spatial manipulation of numbers. This type of representation is related to the temporo-occipital cortex of both cerebral hemispheres. The first code, namely the analogical representation of quantities, is the equivalent of the approximate number system, while the second and the third codes can be assimilated with the exact number system. This perspective supports the idea of modularity of the cognitive architecture for verbal and nonverbal processing. The above metioned perspective is also in agreement with the differrent hemispherical specialization. Data colected from split-brain cases of patients with lesions of the corpus callosum revealed an exclusive localization in the left hemisphere of the verbal processing, such asnaming digit or calculating, but a bilaterally localization of nonverbal processings, such as comparing quantities. Therefore, visual, verbal, and quantitative representations of numbers are located in the left hemisphere. The right hemisphere, although it has access to visual and quantity representations, seems to lack a verbal representation of numbers, as well as procedures for exact calculation. Yet, the only calculation ability available to the right hemisphere is approximation. (Dehaene, 2000).

Specific learning disability with impairment in mathematics

When a learner encounters difficulties in acquiring the basic mathematical skills, this condition represents a learning disability called dyscalculia (Raja & Kumar, 2012). Dyscalculia occurs in a context of average intellectual capacity and adequate environmental opportunities (Archibaldet. al, 2013). However, „in the5th revision of the DSM (APA, 2013) the specific developmental disorders of scholastic skills have been grouped together as dimensions within a single category, in each of which there can be a greater or lesser degree of impairment; the psychometric demonstration of a discrepancy to otherwise normal intelligence is no longer required. This change has been introduced to take better account of the heterogeneity of these disorders with respect to performance profiles and comorbidities, and thus to improve the clinical utility of the diagnosis." (Kucian, 2016, pp. 5-6). Some authors consider that mathematics learning disability and dyscalculia are synonymous (Mazzocco et al., 2011). Since 2013, once with DSM-V published, the phrase specific learning disability (SLD) with impairment in mathematics tended to replace the old term of dyscalculia, but in the actual scientific literature, both terms coexist. In order to reduce the confusion with consecutive to a cerebral trauma dyscalculia, most authors prefere the phrase developmental dyscalculia to name SLD with impairment in mathematics (Von Aster & Shalev, 2007; Evans et al., 2014; Raja & Kumar, 2012; Shalevet. al., 2000).The incidence of dyscalculia is estimated at 3 – 6 % of the population, equally affecting the both sexes (Shalev et. al., 2000). Despite the fact that children in the lower decile of math proficiency tests only fall into the category of those with developmental dyscalculia, about a quarter of school childrenencounterSLDwith impairment in mathematics (Murphy et al., 2007, apud. Kucian, 2016).

Developmental dyscalculia is a heterogeneos disorder (David & Roșan, 2017; Kucian, 2016), including impairments of the number sense, difficulties in memorizing arithmetical facts or in reasoning. SLD is frequently associated with comorbities (alexia, agraphia, attention deficit with hyperactivity disorder, impairments of executive functions), suggesting that multiple brain dysfunctions or dysfunction in central brain areas cause the appearence of developmental dyscalculies (Kucian, 2016).Butterworth (2003) emphasised that children with dyscalculia have difficulty with or simply cannot (a) count and understanding the one to- one correspondence between numbers and objects, (b) estimate or compare numbers and quantities, (c) tell time, (d) learn and remember basic math facts, (e) do mental math, (f) learn math concepts including multiplication tables, formulas, and rules, (g) budget money and balancing checkbook (apud Soares&Patel, 2015). When the nature of difficulty is considered as criterion, seven types of dyscalculia can be identified: (1) verbal dyscalculia – inability to verbally designate mathematical terms and relationships, (2) practognostic dyscalculia – incapacity to manipulate concrete or graphically illustrated objects, (3) lexical dyscalculia – inability to read mathematical symbols and their combinations, (4) graphical dyscalculia – inability to manipulate mathematical symbols in writing, (5) ideognostic dyscalculia – inability to understand mathematical terms and relations and to calculate mentally, (6) operational dyscalculia – inability to carry out mathematical operations, and (7) sequential dyscalculia. (Sharma a&nd Loveless, 1986, apud. Hugheset. al., 1994; Raja & Kumar, 2012). Dyscalculic learner exhibits a group of warning signs in measure to facilitate a correct diagnostic: slowness in giving answers to mathematics questions compared with normal learners, difficulties in mental calculations, using fingers to count simple totals (Green, 2017), mistakes in interpreting word problems, difficulty to remember basic mathematics facts, losing track when counting or saying multiplication tables, difficulty in remembering the steps in a multistage process, difficulties with position and spatial organization (Hannell, 2005, apud. Raja&Kumar, 2012).

One of the paradigms used in the sudies on dyscalculia consisted in tasks of comparing symbolic (arabic digits) or non-symbolic stimuli (dot patterns, Kucian, 2016). The non-symbolic dot comparison task is considered to bethe standard task to assess approximate number system acuity (Dietrichet al., 2015). Numerical axis estimation, computation and magnitude comparison tasks have been consistently identified as reliable predictors of math deficiencies in students (Tavassolie, 2011, apud. Soares & Patel, 2015).Whn judging which of two dot arrays is more numerous, the ratio between quantities influences the corectness of the answers (Feigenson et al., 2004).The magnitude effect refers to the fact that adults judge faster which of two arabic digits is larger when the numerosities are small, while the distance effect refers to the fact that it is easier to appreciate which of two numbers is larger when the distance between them is bigger (Moyer & Landauer, 1967).In pairwise numerical comparison tasks, accuracy of the responses increases, while latency decreases with increasing numerical disparity (Brannon & Terrace, 2002). Even 5 years old children already exhibit this ratio dependence in symbolic numerical comparisons (Temple & Posner, 1998). The effect of distance is more evident in young children, especially if the numbers are presented in a symbolic form (Arabic numeral, and not a collection of dots), due to the recent learning of numbers and the recent introduction of numbers in their vocabulary. The effect of distance temperates with age, as the person's mathematical skills are enriched (Halloway & Ansari, 2010, apud. Gullick, 2012).

The aim of the present study is to evaluate the efectiveness of the number sense incase of normal and adapted curriculum (special learning disabilities with impairment in mathematics) pupils. The presence ofmagnitude and distance effects was evaluated too,in four distinct formats (3D objects, 2D figures, verbal and visual Arabic numerical representation numerals), measuring the number of errors and the reaction times in a comparison task.

Hypothesis I: (a). Normal and adapted curriculum pupils register no significant differences on subitization tasks. (b). Normal currriculum pupils register smaller reaction times on subitization task compared to adapted curriculum students.

Hypothesis II: (a) Normal curriculum pupils register a smaller number of errors on 3D (glued buttons), 2D (non-canonic dots),verbal and symbolic (arabic numbers) tasks compared to adapted curriculum students.(b) Normal curriculum pupils register smaller reaction times on the 3D object comparison, 2D object comparison, verbal and symbolic / numbers in arabic form comparison tasks compared to adapted curriculum students.

Hypothesis III: (a) Both normal and adapted curriculum pupils register a bigger number of errors when comparing big numbers in contrast to small numbers comparisons (the magnitude effect).(b) Both normal and adapted curriculum pupils register bigger reaction times when compare pairs of big numbers in contrast to the situation when pairs of small numbers are to be compared.

Hypothesis IV: (a) Both normal and adapted curriculum children register a bigger number of errors in case of items containing pairs of proximal numbers in contrast to items containing distanced numbers (the distance effect). (b) Both normal and adapted curriculum pupils register bigger reaction times to the items containing pairs of proximal numbers in contrast to the items containing distanced numbers.

Method

Participants

The sample consisted of 120 school children and adolescents, aged 8 to 15 years (M=12,26, std=1,85), 60 of them were special educational needs pupils with adapted curriculum (SLD with impairment in mathematics), while 60 were pupils with normal curriculum. To avoid a bias generated by the subjective designation of the best pupils in the class or those preferred by teachers, were invited to participate those pupils with normal curriculum who immediately followed in the catalog after a pupil with adapted curriculum. In case of absence or refusal, the following one was invited, and so on. After data colection, 111 participants only remained, because of technical problems during recording (lack of sound or image, that made impossible calculation of reaction times), and the new distribution was 56 normal curriculum children and 55 adapted curriculum children. Gender distribution was 63,9% males and 36,1% females. 72, 97% of the responents learn in a rural school, while 24,32% learn in an urban school (in case of 5 respondents this information was not available).

Procedure

Corectness and fasteness of response in tasks of comparing glued buttons, non canonic distributed dots, respectively, pairs of numbers presented in verbal and arabic formats, were measured for normal and adapted curriculum pupils.

Twelve pairs of numbers were presented, always the same and in the same order (but in four different formats): (I) 5 vs. 15; (II) 40 vs. 30; (III) 6 vs. 4; (IV) 20 vs. 18; (V) 7 vs. 14; (VI) 22 vs. 17; (VII) 10 vs. 5; (VIII) 4 vs. 7; (IX) 4 vs. 5; (X) 9 vs. 12; (XI) 8 vs. 9; (XII) 21 vs. 28. The experimenter did not provide any feedback on the correctness of the choices made by the participants, but maintained an supportive attitude and offered general encouragement: "Very good!", "You are doing very well!"

Firstly, participants were asked to appreciate which of the sets of dots was more numerous, the whites ones or the blacks, in a serie of sheets in A4 landscape format. Twelve sheets with glued white and black buttons, in a non-canonic format, were presented. The order of the correct answers was: W (white) , W, B (black), B, W, W, W, B, B, B, B, W. Six times the most numerous set of dots was positioned in the left side of the page, while six times it was in the right side. Then, a task of subitizing was administered, consisting of 6 sheets with one, two or three white and black dots to be compared. The third task administered was similar to the first one, but consisting of 12 sheets with white and black dots (without glued buttons). The forth task was the verbal one, and consisted of 12 questions. (Exemple. Item 1: Ana has five roses. Maria has fifteen roses. Who has more?). In order to reduce the errors consecutive to the different reading speeds, examiner read himself the questions. The fifth task consisted of 12 comparisons between pairs of arabic numbers.

In calculating the reaction times, the participant was considered as being exposed to the stimulus when the page above the board was located approximately perpendicular to the table plane, and the time attached to the frame closest to this position was recorded. The minimum distance between frames was 3 hundredths of a second, the limit being imposed by the capacity to discriminate of the VideoPad software used. It was considered as the moment of reaction the beginning of the sinusoid related to the voice of the respondent giving his last answer (in case he wanted to give another answer) or the second frame after the movement of the hand towards the chosen answer became visible. If a respondent provided the answer both verbally and by hand movement (non-verbally), the first detectable response was taken into account. In the case of verbal tasks, it was considered that the respondent was exposed to the stimulus at the end of the sinusoid related to the voice of the experimenter who uttered the last number given for comparison, and not the end of reading the entire text. In fact, in many cases, the respondents chose while the experimenter was still reading the final question. If a respondent answered before the experimenter finished reading the last number offered for comparison by virtue of the higher reading speed in the mind, the reaction time was considered equal to zero. Reaction times were determined as the absolute value of the difference between the recorded reaction time and the exposure time.

Results

Subitization task

Firstly, a comparison between the number of errors encountered by normal and adapted curriculum pupilswas made. Kolmogorov - Smirnov testrevealed that the distribution of data is not a gaussian one neither for the normal curriculum pupils (z =0,536, p <0.001), nor for adapted curriculum pupils. Hence, the non-parametric Mann - Whitney test indicated that there were no significant differences between children with a normal curriculum and those with an adapted curriculum in terms of the number of errors (z = -1, 028, p = 0.304).

In terms of the reaction times, the Kolmogorov - Smirnov test (K - S) indicated a normal distribution for the “normal curriculum” condition ($z = 0.099$, $p = 0.2$), but the reaction times in case of adapted curriculum pupils was not a normal one (z = 0,182, p <0,001). Consequently, the nonparametric Mann Whitney test revealed that, despite the fact that adapted curriculum pupils encountered bigger reaction times, the differences are not statistically significant (z = -1,543, p = 0,123).

Normal – adapted curriculum pupils comparisons

3D (glued buttons) task. Due to the fact that the distributions of errors are not normal, neither in the case of normal curriculum pupils (Kolmogorov - Smirnov z = 0,251, p <0.001), nor for the adapted curriculum pupils (Kolmogorov - Smirnov z = 0,181, p <0.01), the non-parametric Mann-Whitney test was used. The above mentioned test showed that there are no significant differences between the number of errors in the answers given by pupils with normal curriculum and the number of errors in the answers of pupils with adapted curriculum (z = -1,33, $p = 0.184$).

When reaction times were considered, the distribution of data for normal curriculum pupils was not a normal one (z = 0,233, p < 0,001),nor for adapted curriculum pupils (z = 0,308, p < 0,001) Hence, the non-parametric Mann - Whitney test was used.Adapted curriculum pupils registered statistically significant bigger reaction times than their normal curriculum peers ($z = -2.022$, p <0,05).

2D (dot) comparisons. In terms of the number of errors, the normal curriculum pupils' results did not approach a normal distribution (Kolgomorov-Smirnov z = 0,274, p <0.001), nor those obtained by pupils with adapted curriculum (Kolgomorov - Smirnov z = 0,25, p <0.05). Consequently, the non-parametric Mann-Whitney test was used to compare the two groups. The score obtained z = -2,012, p < 0,05, revealedthat pupils with adapted curriculum summed up statisticaly significant more errors than pupils with normal curriculum.

When reaction times were considered, the distribution of data for normal curriculum pupils was not a normal one ($z = 0,131$, $p < 0,05$), nor for adapted curriculum pupils ($z = -1,845$, $p < 0,001$) Non-parametric Mann - Whitney test indicated that, although adapted curriculum pupils registerd bigger reaction times compared to their normal curriculum peers, differences are not statistically significant ($z = -1,845$, $p =0,065$).

Verbal format task. The distribution of errors recorded for pupils with normal curriculum was not a normal one (Kolgomorov - Smirnov indicator $z = 0,304$, $p <0.001$), nor the distribution of the number of errors recorded by pupils with adapted curriculum (Kolgomorov - Smirnov $z = 0,241$, $p <0.001$). Consequently, we used the non-parametric Mann - Whitney test, whose value $z = -2.316$, $p <0.05$, highlighted that adapted curriculum pupils registered significantly more erroneous answers than pupils with normal curriculum.

The distribution of reaction times for normal curriculum pupils was not a normal one ($z = 0,169$, $p < 0,01$), nor for adapted curriculum pupils ($z = 0,226$, $p < 0,001$) Non-parametric Mann - Whitney test indicated adapted curriculum pupils registered bigger reaction times compared to their normal curriculum peers ($z = -2,838$, $p <0,01$).

Arabic format. When the number of errors were taken into account, given that the distribution of the errors was not a normalone neither in the case of normal curriculum pupils (Kolmogorov - Smirnov $z = 0,536$, $p <0.001$), nor in the case of pupils with adapted curriculum (Kolmogorov - Smirnov $z = 0,507$, $p <0.001$), the comparison of the means was performed with the non-parametric Mann - Whitney test. The adapted curriculum pupils group registered a bigger number of errors than normal curriculum pupils ($z = -2,038$, $p < 0.05$)

Regarding the reaction times, the distribution of data for pupils with normal curriculum is a normal one (Kolmogorov - Smirnov $z = 0.068$, $p = 0,2$), but in the case of children with an adapted curriculum the distribution was not a normal one (Kolmogorov - Smirnov $z = 0,208$, $p <0.001$). Consequently, the non-parametric Mann - Whitney test was used to compare the two distributions. Despite the fact that adapted curriculum pupils registered bigger reaction times, the differences were not statistically significant ($z = -1.115$, $p = 0.265$).

*Comparisons between different formats.*Normal curriculum pupils registered fewer errors in the arabic format than in the 3D format (Wilcoxon $z = -4,695$, $p < 0,001$), fewer errors in the arabic format than in the 2 D format (Wilcoxon $z = -5,026$, $p < 0,001$), and fewer errors in the arabic format than inthe verbal format (Wilcoxon $z = -4,325$, $p <0,001$). No significant differences were found between 3D and 2D formats (Wilcoxon $z = -0,96$, $p = 0,337$), nor between 3D and the verbal format (Wilcoxon z = -0,717, p = 0,473), and nor between 2D and the verbal format (Wilcoxon z = -1,668, p = 0,095).

In terms of reaction times, normal curriculum pupils registered significant smaller values in the arabic task compared to the 3D task (Wilcoxon z = -6,275, p < 0,001), smaller reaction times in the arabic task than in the 2D task (Wilcoxon z = -5,992, p < 0,001), smaller reaction times in the arabic task than in the verbal task (Wilcoxon z = -4,681, p < 0,001). Also, smaller reaction times were recorded for the 2D format than for the 3D format (Wilcoxon z = -5,847, p < 0,001), and for 3D format than for the verbal format (Wilcoxon z = - 3,615, p < 0,001). No significant differences were found between 2D and verbal format (Wilcoxon z = -0,729, p = 0,466).

Adapted curriculum pupils encountered fewer errors in the arabic format than in the 3D format (Wilcoxon z = -4,354, p < 0,001), fewer errors in the arabic format than in the 2 D format (Wilcoxon z = -4,971, p < 0,001), and fewer errors in the arabic format than in the verbal format (Wilcoxon z = -4,61, p <0,001). No significant differences were found between 3D and 2D formats (Wilcoxon z = -1,498, p = 0,134), nor between 3D format and the verbal one (Wilcoxon z = -0,857, p = 0,391), nor between 2D format and verbal one (Wilcoxon z = -0,504, p = 0,614).

Adapted curriculum pupils registered significant smaller values in the arabic task compared to the 3D task (Wilcoxon z = -6,140, p < 0,001), so than the 2D task (Wilcoxon z = -5,305, p < 0,001), so than the verbal task (Wilcoxon z = -5,728, p < 0,001). Also, smaller reaction times were recorded for the 2D format than for the 3D format (Wilcoxon z = -5,192, p < 0,001), and for 2D format than for the verbal format (Wilcoxon z = - 2,828, p < 0,001). No significant differences were found between 3D and verbal format (Wilcoxon z = -0,766, p = 0,444).

The magnitude effect

In order to verify the presence of a size effect when pairs of numberswere compared, single digit numbers were considered as small numbers (the pairs III, VIII, IX and XI, see Procedure section), while the numbers consisting of tens and units were considered as big numbers (the pairs II, IV, VI and XII).

3D (glued buttons) task. In the case of normal curriculum pupils, the data distribution was not a normal one, neither for the pairs of small numbers (Kolmogorov - Smirnov z = 0,517, p <0.001), nor for the pairs of large numbers (Kolmogorov - Smirnov z = 0,301, p <0.001). The non-parametric Wilcoxon test showed that students with normal curriculum had significantly more errors when they compared pairs of large numbers than when they compared pairs of small numbers (z = -4,204, p <0.001).

The distribution of the average reaction times for the pairs of small numbers was not a normal one (K - S z = 0.147, p < 0.01), nor the distribution of the reaction times for the pairs of large numbers (K - S z = 1.232, p = 0.96). The non-parametric Wilcoxon test indicated, in case of pupils with normal curriculum, significantly bigger reaction times when participants were exposed to pairs of small numbers compared to the situation when they were exposed to pairs of big numbers(z = -3,886, p <0.001).

For the pupils with adapted curriculum, the distribution of the number of errors recorded when were presented for comparison pairs of small numbers was not a normal one(K - S, z = 0,512, p <0.001), nor the distribution of the number of errors recorded when pairs of large numbers were presented for comparison (K - S z = 0,247, p <0.001). The non-parametric Wilcoxon test indicated the occurrence of a significantly bigger number of errors when the numbers presented were large compared to the situation when they were exposed to pairs of small numbers (z = -4,469, p <0.001) .

The distribution of reaction times, in case of exposure to pairs of small numbers, was not a normalone (Kolmogorov - Smirnov z = 0,232, p < 0.001), nor the distribution of reaction times recorded when participants were presented with pairs of large numbers were (Kolmogorov - Smirnov z = 0,378, p <0.001). The Wilcoxon nonparametric test showed that significantly bigger reaction times were registered when pupils with adapted curriculum were asked to compare pairs of small numbers than when they had to choose which is larger in pairs of big numbers (z = -3,448, p <0.01).

2D (dots) task. In case of the normal curriculum pupils, the distribution of the number of errors was not a normal one neither for the pairs of small numbers (Kolmogorov - Smirnov z = 0,435, p <0.001), nor for the pairs of large numbers (Kolmogorov - Smirnov z = 0,286, p <0.001). The non-parametric Wilcoxon test revealed that a bigger number of errors were recorded in case of pairs of large numbers than in the case of pairs of small numbers (z = -3,089, p <0.01).

The distribution of the average reaction times for the pairs of small numbers was not a normal one (K - S z = 0.165, p < 0.01), nor the distribution of the reaction times for the pairs of large numbers (K - S z = 0,145, p <0,01). The non-parametric Wilcoxon test indicated no significant differences between reaction times recorded for the pairs of small numbers and the pairs of big numbers(z = -1,359, p =0,174).

In case of adapted curriculum pupils, the distribution of the number of errors was not a normal oneneither for the pairs of small numbers (KS z = 0,478, p <0.001), nor for the large numbers (KS z = 0,221, p <0.05). The Wilcoxon test indicated that the number of errors recorded when pairs of big numbers were presented was significantly bigger than when pairs of small numbers were presented ($z = -4.864$, $p < 0.001$).

The distribution of the average reaction times for the pairs of small numbers was not a normal one (K - S z = 0.213, $p < 0.001$), nor the distribution of the reaction times for the pairs of large numbers (K - S z = 0,163, p <0,01). The non-parametric Wilcoxon test indicated no significant differences between reaction times recorded for pairs of small numbers and the reaction times recorded for the pairs of big numbers (z = -0,28, p =0,78).

Verbal formattask.In the case of the normal curriculum pupils, the distribution of data was not a normal one neither for the pairs of small numbers(Kolmogorov - Smirnov z = 0,448, p <0.001), nor for the pairs of large numbers (Kolmogorov - Smirnov z = 0,445, p <0.001). The non-parametric Wilcoxon test showed no significant differences between the number of errors encountered in the small numbers condition and those recorded in the big numbers condition (z = -0.022, p = 0.983).

The distribution of the average reaction times was not a normal oneneither for the pairs of small numbers (Kolmogorov - Smirnov z = 0,246, p <0.05), nor for the pairs of big numbers (K - S z = 0,218, p< 0.05). The Wilcoxon nonparametric test highlighted no significant differences between the pairs of small numbers and the pairs of big numbers (z = -1.144, p = 0.252).

In case of pupils with adapted curriculum, the distribution of the number of errors was not a normal one neither for the small numbers (Kolmogorov - Smirnov, z = 0,33, p <0.001), nor for the big numbers (Kolmogorov - Smirnov z = 0,47, p <0.001). The Wilcoxon non-parametric test indicated that a statisticaly significant bigger number of errors were registered in case of the pairs of small numbers than in the case of big numbers (z = -3,137, p < 0.01).

The distribution of reaction timeswas not a normal one,neither for the pairs of small numbers(Kolmogorov – Smirnov z = 0,208, p < 0,05), norfor the pairs of big numbers (Kolmogorov – Smirnov z = 0,315, p <0,05). Wilcoxon nonparametric test indicated significantly bigger reaction times for the small numbers than for the big numbers (z = -2,499, p < 0,05).

Arabic format task. In case of normal curriculum pupils, the non-parametric Wilcoxon test indicated no significant differences between small numbers condition and large numbers condition (z = 0, p> 0.05).

The distribution of average reaction times wasa normal one both for small numbers (Kolgomorov - Smirnov z = 0.1, p = 0.2) and for big numbers (Kolgomorov - Smirnov z = 0.064, p=0,2). The paired samples t test highlighted bigger reaction times in the case of big numbersthan for pairs of small numbers (t (51) = -2,457, p <0.05).

In case of adapted curriculum pupils, the Wilcoxon test indicated that there are no significant differences between the number of errors recorded in the pairs of small numbers and the number of errors recorded in case of big numbers ($z = 0$, $p > 0.05$).

The distribution of average reaction timeswas not a normal one neither for the small numbers (K - S z = 0,202, p <0.001); nor for the big numbers (K - S z = 0,175, p <0.001). Wilcoxon test showed significant differences, in that the average reaction times for pairs of small numbers were bigger than the average reaction times in case of pairs of big numbers (z = -3.435, p <0.01).

The distance effect

In order to verify the existence of a distance effect, the pairs III, IV, VIII, IX, X, XI (see Procedure section) were included in the proximal numbers category, while the pairs I, II, V, VI, VII and XII were considered as distanced numbers. In the small distance condition, the differences between the numbers were 1, 2, respectively 3, and in the big distance condition the differences were 5, 7, respectively 10. However, to exclude the possibility of an interference with the effect of magnitude, those pairs of numbers that have already been statistically processed for the purpose of testing the magnitude effect were removed from the statistical processing. Only the pairs IV, VIII and X for the proximal numbers condition, and the pairs I, V and VII for the distanced numbers condition were kept.

3D (glued buttons) task. In case of normal curriculum pupils, the distribution of the number of errors was not a normal one neither for the proximal numbers (K - S z = 0,405, p <0.001), nor for the distal ones (K - S z = 0,535, p <0.001). The non-parametric Wilcoxon test revealed significant differences, in the sense that more errors appeared in case of the pairs of proximal numbers than in the case of distanced numbers (z = -3.5, p <0, 001).

Average reaction times, in the proximal numbers condition, were not normal distributed (K-S z = 0,254, p <0,001), nor the distribution of average reaction times for distanced numbers (K - S z = 0,169,p> 0.01). The non-parametric Wilcoxon test was used. No significant differences between the average reaction times recordedfor proximal numbers and that recordedfor distanced numberswere highlighted (z = -1,270, p = 0.204).

In case of adapted curriculum pupils, the distribution of the number of errors recorded in the proximal numbers condition was not a normal one (K - S z = 0,361, p <0.001), nor the distribution of the number of errors when distanced numbers were compared (K - S z = 0,536, p <0.001). The Wilcoxon nonparametric test indicated that significantly more errors occured in the proximal numbers condition compared to the distanced numbers condition (z = −4,104, p <0.001).

Average reaction times recorded for the pairs of proximal numbers were not normal distributed (z = 0,356, p < 0.001), nor the distribution of the reaction times for the distal pairs of numbers (z = 0,226, p <0.001). The non-parametric Wilcoxon test indicated that average reaction times in the case of proximal pairs are bigger than in the case of distal pairs (z = - 2,803, p < 0.01).

2D (non canonic dots) task. The distribution of the number of errors for the pairs of proximal numbers, in case of normal curriculum pupils, is not a normal one (K - S z = 0,455, p <0.001). The Wilcoxon nonparametric test indicated that significantly more errors appeared in the proximal number condition than in the distal number condition (z = -3,419, p < 0.01).

The distribution of average reaction times in the proximal numbers condition was not a normal one (K - S z = 0.137, p< 0.05), whilethe distribution of the average reaction times for the pairs of distanced numbers was a normal one (K - S z = 0.084, p=0,2). Wilcoxon test highlighted statistically significant bigger reaction times for distal pairs of numbers than for proximal numbers (z = -4, 294, p <0,001).

The distribution of the number of errors recorded when adapted curriculum pupils were exposed to pairs of proximal numbers is not normal (K - S z = 0,409, p <0.001), nor the distribution of the number of errors in distanced numbers condition (K - S z = 0,540, p <0.001). The Wilcoxon non-parametric test indicated a significantly higher number of errors in the proximal number condition than in the distal number condition (z = - 3,662, p <0.001)

The distribution of reaction times measured in the proximal numbers condition, in case of adapted curriculum pupils, is not a normal one (K - S z = 0,221, p <0.001), nor for the distanced numbers (KS z = 0,196, p <0,001). The non-parametric Wilcoxon test indicated that significant bigger reaction times were recorded in distanced numberscondition compared to the proximal numbers condition (z = −3,563, p < 0.001)

The verbal task. In case of normal curriculum pupils, the distribution of the number of errors recorded in the proximal numbers condition was not a normal one (K - S z = 0,449, p <0.001), nor the distribution of the number of errors in the distanced numbers condition (K - S z = 0,477, p <0.001). The Wilcoxon test revealed no statistically significant differences between proximal and distal pairs (z = -0.728, p = 0.467)

In case of normal curriculum pupils, average reaction times for the proximal numbers condition were not normal distributed (K - S z = 0,249, p <0.05), nor the average reaction times recorded when pairs of distanced numberswere compared (K - S z = 0,306, p <0,05). The Wilcoxon nonparametric was used. Although the average reaction times were biggerin the proximal numbers condition than in the distanced numbers condition, differences are at limit statistically significant(z = -1,933, p = 0,053)

In the case of adapted curriculum pupils, the distribution of the number of errors recorded for the pairs of proximal numbers is not normal (K - S z = 0,397, p <0.001), nor the distribution of the number of errors recorded distanced numbers were compared (K - S z = 0,353, p <0.001). The Wilcoxon test indicated no significant differences between the number of errors recorded when the pairs of numbers compared were proximal and the number of errors recorded when the pairs of numbers were distal (z = -0.751, p = 0.452)

In caseof adapted curriculum pupils, the distribution average reaction times in the proximal numbers condition was not a normal one (K - S z = 0,315, p <0.05), nor the distribution of average reaction times for pairs of distanced numbers (K - S z = 0,184, p <0,05). The Wicoxon test revealed that, despite the fact that the reaction times were recorded for proximal pairs of numbers were bigger than the average reaction times for distanced numbers, the differences are not statistically significant (z = -1,843, p =0.065).

Arabic numbers task. In the case of students with a normal curriculum, the distribution of the number of errors for the distal number pairs was not a normal one (K - S z = 0,536, p <0.001). The Wilcoxon test revealed no significant differences between the number of errors recorded in the proximal condition compared to the distal condition (z = -1, p = 0.317).

The distribution of the average reaction times for the pairs of proximal numbers, in the case of normal curriculum pupils, was a normal one (Kolmogorov - Smirnov z = 0.104, p = 0.2), but the distribution of average reaction times in the distanced numbers condition was not gaussian (K - S z = 0,202, p <0.001). The Wilcoxon test reveled bigger average reaction times for proximal pairs of numbers than in the case of distal pairs (z = -6.102, p <0.001)

In case of adapted curriculum pupils, the distribution of the number of errors recorded in the proximal number condition was not a normal one (K - S z = 0,535, p <0.001), nor the distribution of the number of errors in the distanced numbers condition (K - S z = 0,536, p <0.001). The Wilcoxon test indicated no significant differences between the number of errors recorded in the proximal numbers condition and those for encountered in the distanced numbers condition (z = -1.633, p = 0.102).

The distribution of average reaction times for the pairs of proximal numbers, in the case of adapted curriculum pupils, were not a normal one (K - S z = 0,165, p < 0,01), nor the distribution of average reaction times for the pairs of distanced numbers(K - S z = 0,235, p <0.001). The Wilcoxon test indicated that the average reaction times for proximal pairs of numbers are significantly bigger than the average reaction times for the distal ones (z = -5.411, p <0.001).

Discussion

As expected, in the subitization task no differences were highlighted between normal and adapted curriculum pupils with respect to the number of recorded errors but, more than that, no differences were found in terms of reaction times. These findings are in agreementwith the idea that subitization is the simplest and the most effective process of computing comparisons, perhaps the common land of both approximate and exact number systems. In case of the pairs of numbers smaller than 4, both normal and adapted curriculum pupils were able to make comparisons corectly and fastly.

When numbers bigger than 4 were used, differences between the two groups appeared, but these differences were dependingon the format used to present the quantities. Both normal and adapted curriculum pupils registered a comparable number of errors in the 3D (glued buttons task). This somehow paradoxically situation finds an explanation when the reaction times are taken into account. Adapted curriculum pupils need longer time to solve the same task. In the next task, the 2D (non-canonic dots) one, both normal and adapted curriculum pupils register comparable reaction times, but the increase in speed has a price for the SLD pupils with impairment in mathematics, namely an increase of the number of errors.The possible explanations of these differences might be offered in terms of the strategy each group of pupils used in solving comparison tasks. Initially, the experimenter asked the participants to appreciate which are more, with no reference to time limit. The first task was an accomodation one and the corectness of the answers was the aim of the both groups. While a correct answer could be offered only with the usage of the exact number system, SLD pupils need a longer time to compute the comparisons and these might explain the recorded differences of average reaction times. Due to the fact that the use of the exact number system is a tiringapproach, when exhaustion appeared, pupils with SLD with impairment in mathematics sacrificed the corecteness in favour of the speed, appealing to the approximate number system. In the verbal and arabic (visual) tasks, as expected, normal curriculum pupils encountered lesser errors and smaller reaction times than their adapted curriculum peers. Even if considering the reaction times comparable in the arabic task, the fasteness of answers in case of the adapted curriculum pupils might considered as the result of a mecanical memorization. Children use daily arabic numbers, not only in school related activities, and these continous exercise improve their capacity to manipulate numerical quantities.

More interesting for the educational activity is the finding that the pupils, no matter of the type of curriculum, exhibit the highest effectiveness when they use the visual Arabic numerical representation format, and these efectiveness manifests itself not only in fasteness, but in corectness, too. Both normal and adapted curriculum pupils show significant differences between arabic and verbal formats, in terms of number of errors and reaction times. The preference for the arabic format might be the result not just of the ease of operating with icons than with words, but it could derive, too, from the fact that digits are acquired before the letters and the written language and digits have a sense by themselves (Petrovici, 2014). The warning signal for the educators is that, during the math classes, prolix, unclear, useless phrases should be avoided. On the other hand, in terms of number of errors, no differences appeared between the 3D and the 2D formats, neither for normal curriculum pupils, nor in case of the adapted curriculum pupils. More than that, the 3D format seems to be associated with longer reaction times in case of SLD pupils. The use of concrete objects during the math class in the detriment of the images projected on a display, unless the lack of the adequate devices, is not justified.

A magnitude effect was revealed in both normal and adapted curriculum children, in the 3 D task, when the number of errors was taken into account. In terms of reaction times, paradoxically, bigger values were encountered for the pairs of small numbers than for the pairs of big numbers, in case of both normal and adapted curriculum pupils. The possible explanation of this finding might be the fact that both groups of pupils are more likely to use exact calculation strategies when faced with small numbers and approximate calculation strategies when confronted with big numbers. The fact that the magnitude effect is present, in case of the adapted curriculum pupils, in the 2D task with an identical reaction time pattern, seems to sustain the above formulated explanation. In case of normal curriculum pupils, in the 2D task, the magnitude effect is no more visible, nor in terms of number of errors, nor in terms of reaction times. This results might be explained by the higher learning capacity of the normal curriculum pupils, and due to the accomodation with de task, their capacity to choose fastly the correct answers improves and the magnitude effect is no more visible. On the contrary, considering the number of errors, SLD pupils exhibit the magnitude effect in 2D task, but not in the verbal, nor in the arabic tasks. The reaction times were bigger for the small numbers, contrary to the expectations. The explanation might be that, in the verbal and in the arabic tasks, approximate calculation strategies are no more applicable. The normal curriculum pupils encounter no errors, due to an effect of plateau and this plateau is reached by the adapted curriculum pupils in the arabic task, too. But in the verbal task, the results against expectations seem to highlight a third, different, strategy of the SLD pupils: when confronted with difficult tasks, they choose the answers randomly.

The distance effect seems to be more outlined, both in case of normal and adapted curriculum pupils. As expected, in the 3D and 2D tasks, more errors occured in the cases of proximal than in the case of distanced numbers. In terms of the number of errors, in the arabic visual task no differences emerged between the two groups of pupils, probably due to an effect of plateau, as explained below.In terms of reaction times, the distance effect emerged: both normal and adapted curriculum pupils registered bigger reaction times in the case of pairs of proximal numbers than in the case of pairs of distanced numbers.In the verbal task, no differences between proximal and distanced numbers, in terms of number of errors were found neither for normal curriculum pupils, nor for adapted curriculum ones. More than that, no differences emerged in terms of reaction times. The possible explanations seem to be different for the two groups: the normal curriculum pupilsresponded fastly and correctly to a easy task for them (the plateau effect), while the adapted curriculum pupils reacted slowler and gave more answers at random.The fact that in the 2D task both normal and adapted curriculum pupils registered bigger reaction times in the case of the distanced numbers than in the case of the proximal numbers sustain the existence of the distance effect in terms of reaction times, but this finding rise an obstacle in explaining the differences between normal and SLD pupils through the different calculation strategies. A smaller reaction time in case of the pairs of distanced numbers were expectable, due to the use of an approximate calculation strategy. Taking into account that for the same level of performance, children with SLD engage more intensely visual working memory, attention and selection of responses (Kucian, 2016), the clue is perhaps at an another level. Choosing one strategy and not the other is determined based on the the stimuli presented and the behaviour required (Feigenson et al., 2004).Emotions and, especially, mathematical anxiety, play an important rolein the decision-making process about the strategy to use, .

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