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Phd Thesis

The Implementation of GeoGebra Program into Pre-Service Mathematics Teachers training in Israel

Abstract

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Dedication

First and foremost, I would like to dedicate the body of this work to my supportive and loving mother, Fatmeh Husein. Without her, I would not be where I am today, nor would I be what I am today, and none of these things would have been possible. I highly appreciate your endless efforts in bringing up me and my thirteen siblings after my father passed away 38 years ago. Your wisdom, kindness, and unconditional love lit my way throughout my whole life.

Acknowledgment

I would like to take advantage of this opportunity to thank my dedicated and supportive supervisory professor Carmen Cretu for her continuous support and guidance throughout the whole process of creating this dissertation. She has given me the deep understanding and insight to better analyze, form, and study the subject matter and therefore I am certainly deeply grateful for that. Moreover, many thanks to the remarkable committee members for their insightful comments, remarks, and suggestions throughout the creation of this dissertation.

I also want to sincerely thank my devoted student Doaa Aboleil for her help and assistance in the process of the research as well as to the professional Egal Sapir for his outstanding contribution in enriching my implementation of GeoGebra software.

Content

Introduction

This study is an attempt to bridge the gap between traditional methods of math education and more innovative math teaching methods through technological integration in pre-service math teachers' curriculum and field experience using GeoGebra software.

Why GeoGebra?

More than 10 academic and non-academic studies are published every year in the research area of GeoGebra integration for math education. However, no studies have been published in Israel dedicated to the use of GeoGebra for teachers' professional development.

Research Aims and Objectives

1) Analysis of the math teachers' professional development process through the introduction and practice of digital technology tools in relation to GeoGebra.
2) To identify of the specific skills which enable pre-Service teachers the ability to successfully integrate technology in Math Education.

General Research Hypothesis

1) Introducing GeoGebra into didactic courses will contribute more to the professional development of pre-service teachers than other, theoretical ways of introducing GeoGebra in educational institutions.

2) Introducing GeoGebra software will positively affect the attitudes and beliefs of pre-service mathematics teachers towards mathematics and math education.

3) The use of GeoGebra software will help develop pedagogical skills of pre-service mathematics teachers.

Thesis Overview

In My dissertation Three Studies have been investigated:

1) Study 1 -Identification of the main challenges in introducing GeoGebra Courses
2) Study 2 -The modification of Pre-Service Teachers' Attitude during GeoGebra Course
3) Study 3- Pre-Service Mathematics Teachers' Ability to Identify Skills Needed to Work with GeoGebra Software.

All the studies use a combination of specific Quantitative and Qualitative tools including observable tools.

Study 1- Identification of the main challenges in introducing GeoGebra Courses

PSTs emphasized the potential benefits of GeoGebra compared to other programs due to "the range" and "ease of use" factors. Four of GG's 21 classic tools were perceived as complex and difficult: Slider, Insert text, Insert image, Rotate the object around the point by an angle. PSTs' comments, suggestions, and characterizations of the issues suggest more than 500 problems, although not all them were relevant or accurate. The participants' feedback revealed that they had difficulties with the syntax of algebraic input and in the use of commands. The latter was rated as more difficult than the use of dynamic geometry tools. Considering the low difficulty ratings for workshop activities, the difficulty level of the course seemed to be appropriate for future secondary school math teachers.

Study 2 -The modification of Pre-Service Teachers' Attitude during GeoGebra Course

Attitude toward mathematics and toward math teaching and a model of how GeoGebra influence on development of mathematical thought were investigated. The Attitude related to GeoGebra use of math pre-service teachers are very positive a priori and after a GeoGebra course. Two different groups of PSTs are recognized those who did not exhibit notable improvement in any way by working with the software (about 10%), and those who improved their attitudes while working with GG (app. 90%).

The data analysis revealed the importance of collaborative work as being the second influential factor after the software in improving attitudes such as self-confidence, Flexibility of Thought, Creativity and Systematization, and the development of communicative competence.
This analysis also pointed to the collaborative work and interaction among PSTs as being the third important factor, after the software and the pre-service interaction with teacher-instructor, to develop thinking and reasoning and argumentation-proof competencies.

The results show an improvement in affective and behavioral components among most participants but indicated no significant improvement regarding the cognitive component.

Study 3 -Pre-Service Mathematics Teachers' Ability to Identify Skills Needed to Work with GeoGebra Software.

The study framework investigated the specific skills require to use GeoGebra in Math teaching and how to teach concepts by multiple representations of concepts**;** Identifying and addressing students' difficulties and misconceptions regarding the new concepts, using built-in methods and strategies to teach concepts and Using tools enabling concept assessment-evaluation.

Results**:** The quality of the first individual projects submitted by the participating PSTs reveal that they aren't aware of the specific skills required to apply when teaching with GeoGebra

Teaching Concepts with GeoGebra

Considering this study's results, one could say that the PSTs did not derive much benefit from this component of GeoGebra, especially with regard to interrelating representations.

An examination of the PSTs' instructional plans for the first projects reveals that they were unable to precisely distinguish the concepts of difficulties, misconceptions, and errors

The most frequent instructional methods and strategies used by PSTs in practice for both the first and second projects were: discovery, direct instruction, discussion, questions and answers, and brainstorming. Data gathered indicates that the PSTs made progress in technological integration of methods and strategies of specific skills, not unique to GeoGebra software, but using various effective technologies in their projects.

Analysis of data obtained in the context of assessment-evaluation component revealed that the PSTs improved their skills in locating relevant information of various technology-assisted tools used for assessment-evaluation and using it in technology-assisted environments for formative and/or summative purposes.

Analysis of the data gathered by qualitative methods indicates that PSTs tend to prefer accomplishments in the curriculum contents involving the statement "information and communication technologies can be benefitted from" to achieve technological integration.

Conclusions:

Teachers cannot be expected to begin successful teaching without the use of specialized software for teaching mathematics.

A "GeoGebra" course should be integrated into the pre-service teachers' curriculum.

It is recommended to increase the use of mobile phones in the workshop and while preparing tasks (GeoGebra on mobile phones).

1. The main dimenssions of Literature review in the frame of thesis

Using GeoGebra (Dynamic Mathematics Software)

GG is an example of a dynamic mathematics software (Hohenwarter, 2008). Insofar as GG, different representations of the same mathematical object connect dynamically, give users the freedom of movement back and forth, thus making the relations between the representations more easily comprehensible (Ozgun-Koca et al., 2010) .

The software offers two representations of each object: *the numeric, algebraic component displays, are used to coordinate an explicit or implied equation, *a parametric equation, while the geometric component assembly shows the corresponding solution (Hohenwarter, 2006) .

Teaching math with GG: GG may be used for presentation, create instruction material, such as notes or interactive (shared) worksheets (Hohenwarter 2005; Fuchs and Hohenwarter, 2005).

Barriers and Difficulties faced by in-class Technology Integration

There is an abundance of literature identifying barriers to technology integration that provides insight into the barrier faced by secondary school teachers of mathematics who use technology as part of classroom instruction (An & Reigeluth, 2011; Drijvers, Doorman, Boon, Reed & Gravemeijer, 2010; Pierce & Ball, 2009; Wachira & Keengwe, 2011) .

Pre-Service Teachers Attitude and Beliefs

Literature has generally concluded that attitudes consist of cognitive, affective, and behavioral components (Crano & Prislin, 2006). This 'tripartite' view of attitude was the starting point for this study. Several studies examined relationships between the different components of teacher attitudes towards math in general (Nisbet, 1991) .

Pedagogical Beliefs: Novice secondary school mathematics teachers possess adequate information from their own experience as learners, including deeply held beliefs of teaching and learning affecting the way they think, approach, and learn mathematics (Darling-Hammond, 2006; Philipp et al., 2007). Ertmer et al. (2012) identified previous beliefs to have the greatest barring influence even more so than first-order barriers to technological integration in the classroom (e.g., the barriers of access and support).

Influence of Previous Schooling: Modeling of technology integration occurs long before pre-service teachers begin the teacher education program. Koch, Heo, and Kush's (2012) findings indicate that pre-service teachers who described their high school experience as being well integrated with technology reported they would be most likely to integrate technology into their future practice.

Today, many pre-service teachers are digital natives since they have the advantage of growing up with technology (Lei, 2009). Some experts suggest that as these digital natives have been immersed in technology from an early age, it is safe to assume that they are fluent in technology (Prensky, 2001; Tapscott, 2009). While the concept of digital native is appealing, evidence is lacking to support these claims (Jones & Czerniewicz, 2010; Lei, 2009).

Changing Teachers' Beliefs: Most researchers agree that past experiences make it difficult for teachers to change their beliefs (Ertmer, 2005; Pajares, 1992) .

Because teachers' beliefs and practice are continually influenced by other people's values, Ertmer and Ottenbreit-Leftwich (2010) propose ways that might help facilitate a shift in teachers' beliefs .

Teachers' Attitudes regarding Math and Math Education: The importance of teachers' attitudes towards mathematics has been researched and documented. Philippou & Christou (1998), and Wilkins (2008) linked teachers' attitudes toward mathematics to their teaching classroom practices. Despite the importance of personal attitude, many pre-service teachers enter the profession with negative feelings towards the subject (Nisbet, 1991; Philippou & Christou, 1998) .

Categories required for math learning and math teaching: selected attitudes identified in most of the literature, considering they constitute a minimum of desirable mathematics attitudes of any PST: Flexibility in Thinking (FT), A spirit of Critical Thinking (EC) , Staying the course (PE) , Focus and Effort r (PR) , Creative Expression (C) , Stand Alone Spirit (AU) and Systematized Approach (SS)

Measuring Integration of Technology in Education

Integration of technology in teaching mathematics provides a very active field of innovative educational technology. Vast literature on this subject provides a broad scope of theories, methodologies, and interpretations, related to educational potential of fresh technologies to aid mathematics instruction.

TPACK Framework Assessment: TPACK, as a knowledge domain, is still in its early, developmental stages regarding its application and reliability of evaluated feedback. Measures include self-evaluation, questionnaires and interviews, logs, reflective journals, diaries, classroom observations, and teaching artifact projects evaluation (lesson plans, student work, classroom activities, and teaching materials). According to Koehler et al. (2012), TPACK reliability and validity tests for currently available TPACK measurement tools are indeed limited. TPACK has a complex nature, the various content areas to be included, different target groups (e.g., experienced and prospective teachers, etc.), and the rapid-growing development of digital technologies. Lyublinskaya and Tournaki (2012) measured TPACK levels rubric based on the four components of TPACK (Niess, 2011), the five levels of the TPACK development model (recognizing, accepting, adapting, exploring and advancing) (Niess et al., 2009), and the Principles for a Practical Application of TI- NSpired technology since it is a content-specific form (Dick & Burrill, 2009). Researchers have used the rubric to analyze teacher artifacts; however, it can also be used for direct evaluation as an observation protocol.

The rubric has strong face validity, but reliability and validity analyses for this newly developed rubric is still developing (Lyublinskaya & Tournaki, 2012) .

Concepts Instructional Practice: Although researchers recognize the potential of having students use computers for exploring mathematics, many teachers have not incorporated technology beyond the use of common calculators (Zbiek, Heid, Blume & Dick, 2007). Research also shows that although many math teachers report that they integrate technology, most of them often report low technology usage and minimal use of drill- and-practice software (Purcell et al., 2013). High-level technology use is typically associated with student-centered or constructivist practices (An & Reigeluth, 2011). Technology-related classroom management is a challenge for pre- service teachers and in-service teachers alike (Hew & Brush, 2007; Lim et al., 2003).

Procedural Knowledge vs. Conceptual Knowledge: Ma's (1999) and other researchers observed that teachers focused on teaching rules and procedures in math, while teachers in countries whose students get higher math scores spent more of their teaching time making connections between math topics and concepts. Researchers have documented the same limited content knowledge among pre-service teachers (Ma, 1999; Philipp et al., 2007) .

Self-Evaluation: Self- evaluations provide questionably validity and accuracy data and problems caused by the teacher's inflated perspective of subjective information (Strong, 2011).

As technology evolves and becomes more prevalent in schools, there is an increasing demand for teachers to develop technology-based lessons appropriate to 21st-century skills. Thus, pre-service teachers must be adequately trained and master the practical knowledge and dispositions required to integrate technology in mathematic lessons.

2. Research General Design

Description: the main portion of the Action Research studies begins with preliminary analysis and continues with an ongoing analysis of the data gathered after each intervention, which will be performed according to the intervention sequences.

The Main Studies (Studies 1 and 2) were conducted on pre-service teachers attending three introductory technology courses.

The Integrated Course Study (Study 3) can also be defined as an action research

The entire work was conducted in the action research (AR) paradigm as part of my interest in transforming research practices. The research method selected was a non-experimental analysis (action research approach) that will be conducted by me as the leading researcher working as an instructor in Sakhnin College for Teachers Education, with the help and collaboration of the research participants.

In all the studies, quantitative and qualitative data analyses (observable and non-observable) were used. Selected PST performance data were analyzed in greater depth. GeoGebra Course

During the 2016-2017 and 2017-2018 academic years, I designed, conducted, and evaluated three “GeoGebra” courses in workshop form for PSTs.

Course description

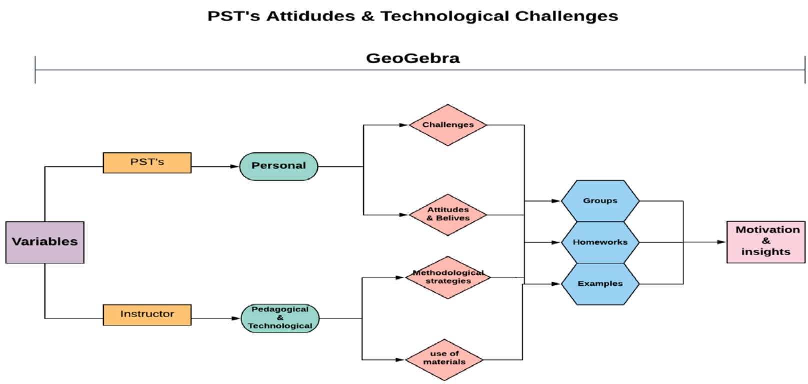
The following section provides an overview of workshops evaluated in this study and presents the tools and features introduced into each workshop.

The central part of each session comprised of four activities designed to help participants obtain the necessary skills for independent use of GG: Introduction to GeoGebra Workshop, Workshop I: basic geometric, Workshop II: angles, transformations, and images' insert, Workshop III: equations, Workshop IV: functions and images export.

2.1 Research Model

The variables investigated in GeoGebra courses are difficulties on the use of the software and the attitude change toward math when the GeoGebra Course is the context of the two studies

Figure 2-1 GeoGebra Course Pre-Service Challenges -Attitude Research Model



The data analysis design was inspired by the O'Cathain et al. (2010) model: a quantitative and parallelly a qualitative analysis from many sources. The triangulation process the integration of the two kinds of data was done in an iterative way.

Studies of GeoGebra Courses – Pre-Service Teachers' Profiles

The Studies participants were 80 PST's in their 2nd to 4th years of study (three courses) pursuing their mathematics teaching degree at the Sakhnin College for Teacher Education.

The first course was comprised of individuals of normal academic performance; they were not brilliant but also not deficient in terms of achieving a positive evaluation in mathematics, with several exceptions. It was a very talkative group; the PSTs were easily distracted and had difficulty maintaining their attention on the tasks at hand. Most of them were not used to investing time to do homework outside of school. Therefore, the researcher hoped that with the incorporation of GG, they would become better motivated towards the subject, show greater dedication, and work harder, which would lead them to better performance.

The participants of the second course behaved similarly to those in the first group.

In the third course, the ratio of participants with cognitive difficulties was higher. The performance of this group of PSTs was relatively poor due in part to a partially unstable and relatively prolonged work rate.

Note: Study 3 Skills for Geogebra use was done in the context of one experimental pedagogical course. The research design was so unique that I decided to explain it in the chapter itself.

The shared characteristics are that all the studies are Action research and the use of qualitative and quantitative tools

Accord on the research design in the next section, I will present the Study 1: Identification of the main challenges in introducing GeoGebra Courses

3. Study 1. Identification of the main challenges in introducing GeoGebra Courses

This study is an attempt to bridge the gap between traditional math teaching methods and innovative math teaching methods through technology integration in pre-service math teachers' (PSTs) curriculum and field experience using GeoGebra software.

3.1. Research Aims

Research Questions

The study seeks to answer the following questions:

1) What are the most important attributes to include in a dynamic geometry software, according to the participating PSTs?

2) What are the barriers/obstacles of using GeoGebra as a teaching aid assistive tool for teaching math, as perceived by PSTs?

3) What standard learning/teaching difficulties occur in the introduction process of GeoGebra?

4) What are the most challenging tools and features of GG for PSTs?

The following more focused supplementary questions were created in order to identify the difficulties the participants encountered during **GeoGebra workshops** when they were introduced to GG, or when they were asked to evaluate its usability:

1) Are the design, content, and difficulty levels of the introduction to technology course appropriate for PSTs trained to teach math in secondary schools?
2) How do PSTs experience using GeoGebra software, and what feedback do they give regarding its usability?
3) Which difficulties and problems occur, and what questions we're asked most often?
4) Do activities with tools relating to dynamic *geometry* influence their difficulty ratings?
5) Do GG features, like algebraic input, or commands cause additional difficulties when introducing the use of GeoGebra?
6) Can GeoGebra's dynamic geometry "classic" tools be classified of common characteristics, determining their general difficulty levels?

3.2. Participants, Method, Finding and Discusion

Participants: The Study participants were 80 PST's in their 2nd to 4th years of study (three courses) pursuing their mathematics teaching degree at the Sakhnin College for Teacher Education

Method: Action Research

Research Tools: The study used tools to analyze non-observable and observable behaviors. These tools were adopted from other studies or designed by the researcher. The following tools, adopted from other studies, were used to determine the competence of the PSTs with GG and sought non-observable information: 1- surveys/feedback questions used before and after every workshop 2- interviews and audio recordings utilized during tasks 3- GG files used during the workshops 4- suggestions by PSTs after the workshop. The following tools, designed by the researcher, were used to determine the competence of the PSTs with GG and sought observable information: 1- skills observation used during the workshop and 2- journals used between tasks. A final tool, designed by the researcher, was used to observe attitudes of PSTs during tasks and was the only tool that did not collect data about PSTs competence with the tool.

Main results

A series of questionnaires were utilized to understand the essential attribute, according to the PSTs, for the GG software. PSTs were asked their opinion regarding 1- Language Attribute 2- Visual Attribute 3- Users' Guide Attribute 4- Most essential property overall 5- Most essential purpose overall.

Most essential language property: Responses to this question revealed that the majority of PSTs believed that the program being in the users' native language is the essential language property (54.8%) followed by language that is appropriate for the curriculum (19.4%).

Most essential visual attribute: According to the majority of PSTs, the most important visual property was screen clarity and readability (29%). The use of the display area was rated as most essential by 18.9% of PSTs.

Most essential users' guide attribute: 24.2% of the PSTs said the user manual being detailed was most important. Likewise, 24.2% said that the manual having a detailed explanation of how to use GG in the help menu was most important. 17.4% of the PSTs said that the manual having a number of examples was most important.

Most essential property overall: 35.4% of PSTs rated ease of use as the most important overall property of GG. 16.9% voted that the program having additional tools, such as spreadsheets and a computer algebra system, was the most essential. 16.8% said the most important property was that the program still allowed the computer to run fast.

Most essential purpose of software usage: 39.1% of PSTs said the most important function of such software is to present the subject matter in the lesson. 16.4% said it was to give students materials to study at home.

Overall, the most essential attribute, according to pre-service teachers, are: being in the users' native language, screen clarity and readability, and detailed user manual or help menu.

PSTs evaluation of GeoGebra features and tools: PSTs' ratings of GeoGebra characteristics in the workshops were collected and the mean ratings were found. Items that had the highest mean score were: 1- the Grid (M=3.71) 2- Point Capturing (M=3.73) 3- Labeling Objects (M=3.7) and 4- Creating a point on an object (3.71). The remaining 11 items' mean scores were between 3.12 and 3.69. The overall mean rating of GeoGebra's characteristics (15 total) was 3.53; thus, PSTs perceived the tools and features of GeoGebra to be user friendly.

Difficulty levels of GG tools: More than 25 dynamic geometry tools were introduced within the framework of the Introduction to Technology Course: GeoGebra.

GeoGebra tools were classified it into one of three difficulty level groups: 'easy to use,' 'medium,' and 'difficult to use.' The critical value for 'easy to use' tools was set at easy = 0.99, which yielded an interval width of 0.51 between the most accessible tool rating and the threshold for this group. 4 of the 21 tools fell into this category, or 20.05% of the tools. The critical value for 'medium difficulty' was set at medium = 1.35, which yielded an interval width of .35. 10 of the 21 tools were categorized as 'medium', which is 46.42% of the tools. The critical value for 'difficult to use' was set at difficult = 1.71, which yielded an interval width of .35. 7 of the 21 tools fell into this category, which was 33.53% of the total.

Complexity criteria of dynamic geometry tools: GG tools were given a complexity rating of 1 to 5, which reflect the difficult of the characteristics of said tools. Criteria was based on initial difficulty level rating as well as on analysis of GG tools. Three tools were given a complexity rating of 1, Five tools were given a complexity rating of 2, Five tools were given a complexity rating of 3, Four tools were given a complexity rating and finally, four tools were given a complexity rating of 5: 1- Slider 2- Insert text 3- Insert image and 4- Rotate the object around the point by an angle.

GG features rating: The participants rated 11 introduced GG features on a scale of 0 'very easy' to 5 'very difficult'. The 11 features were organized into the three difficulty groups. The Easy Group (range of .68 – 1.20). The Middle Group (range of 1.21 to 1.62). The Difficult Group (range of 1.63-2.05) is made up of: 1- Background Image 2- Redefine and 3- Auxiliary objects. Background Image is accessed by the Properties Dialog; Redefine is accessed by the Context Menu; and Auxiliary Objects is accessed by the Properties Dialog.

Menu bar features: GG's menu bar comprises of seven items that each of them can be directly selected due to a sub-menu enabling access to certain features and setting change.

Navigation bar features: This information is essential for using GG by mobile phones, but it challenges the analysis of the workshop context. The Navigation-bar features also include Construction Protocol and Context Menu features.

Dialog properties features: The dialog Properties of GG can be accessed in three different ways: selecting the target feature from either the Edit menu or the Context menu or double-clicking on an object in Move mode.

General reporting review: Although not all the PST reports were relevant or accurate, more than 500 problems were noticed and reported during the 3 courses examined. These problems can be categorized into six different areas: 1- Problems with mathematical content 2- Computer issues 3- Problems with GeoGebra in general 4- Problems with GeoGebra Tools 5- Problems with GeoGebra Features and 6- Problems with Algebraic Input and Commands.

Mathematical content: summarizes feedbacks of mathematical contents within the framework of the workshop's activities. The word 'activities' was coded to determine the number of reported problems (75) with mathematical content.

Computer issues: summarizes general computer issues that occurred during the introduction to workshops. The words 'files' and 'input device' were coded to determine the number of reported computer issues. A problem with 'file' was reported 18 times and 'input device' 9 times.

GG in general: summarizes general GG issues and characteristics participants had difficulty to master or cope with. Four words were coded to categorize general problems: basic handling (10); GeoGebra concepts (52); Selecting objects (26); and other issues (25).

GG features: summarizes feedback of GG features that participants found as challenging and, therefore, may require revision of the teaching approach. 'Menu bar', 'properties dialog', 'context menu', 'other features' were used to code a complaint under GG features. 'Menu bar' had a frequency of 32 complaints; 'properties dialogue' had a frequency of 42; 'context menu' had a frequency of 33; and 'other features' showed up 22 times.

Algebraic input and commands: summarize the helper's feedback regarding algebraic input and the use of pre-defined commands in GG. 'Input Syntax' and 'Use of commands' were the words used to code a problem under this category. A problem with 'Input Syntax' was reported 54 times and 'Use of commands' 16 times.

PSTs Expectations and Perceptions of GeoGebra Tool

Course surveys: Findings concerning the PSTs' perceptions of GeoGebra features were moderate. Most PSTs felt that GeoGebra software is user-friendly and relatively easily understood and explored. Similar comments were recorded in Green & Robinson (2009) study.

Difficulties in using GeoGebra tools during the introduction to technology course for construction of basic Geometry: The findings of this study indicate that PSTs' perceptions of features and tools taught in the Introduction to Technology Course were highly positive.

Preiner (2008) reported that these types of workshops tend to be more comfortable for the students and the findings of this study are consistent his.

Difficulties in using GeoGebra tools angle and transformation: Preiner (2008) stated that Activity1, angle transformation, was the easiest in this workshop compared to the other four activities introduced; the findings of his study are consistent with those of this study.

Difficulties in using GeoGebra tools on coordinate and equation: The findings relating to PSTs' perceptions of the features, tools, and commands in this workshop were moderate.

Difficulty observed in workshop in activating present and solve function and perform image export: The findings relating to this workshop, although considered as moderate, were inconsistent with those of Preiner (2008), which indicated that activities in a similar workshop were very natural.

Discussion

The findings of the first study revealed that mathematics PSTs have a priori positive attitude towards the use of GeoGebra. Mathematics, which is an abstract subject, requires students and teachers to collaborate, imagine, and think in areas of geometry and transformations. It is important then to use technology and promote student-centered learning where the teacher acts as an enabler or facilitator; GeoGebra software is expected to help mathematics PSTs to accomplish this.

3.3 Recommendations

Analysis of the first study shows that introducing the open-source mathematics software GeoGebra in an instructional technology course to prospective mathematics teachers helps them to develop a positive predisposition towards the use of technology.

The need for an introduction to technology course for PSTs: Thus, in order to ensure the assimilation of GeoGebra program into preservice teachers' pedagogy, the course, 'Introduction to Technology,' a course combining introduction of innovative educational technology and pedagogy, should be utilized to train PSTs.

GeoGebra Courses in the form of workshops are more effective than frontal methods, as this form enables active learning

The Israeli ministry of education has no unified policy regarding preference to specific educational aids, but yet GeoGebra is recommended because GeoGebra is known in Israel and has been found appropriate for math teaching, The High School Teachers Organization publishes GeoGebra based lessons, there are some groups in the field that are engaged in developing program-based lesson plans.

• The program is also commonly used in Arabic speaking countries.

• The Israeli Ministry of Education has a project aimed at evaluating uses GeoGebra.

Incorporating mobile devices in math teaching: Modern mobile applications contribute to the formation of interactive, dynamic educational environment, which enables the students to implement the exercises, problems, and tasks, and develop their abilities. (Dias & Victor, 2017; Grant, et.al. 2015; Thomas & Muñoz, 2016; Supandi, Ariyanto, Kusumaningsih & An, 2017)

4. Study 2: The modification of Pre-Service Teachers' Attitude during GeoGebra Courses

4.1. Research Aims

Assumption (Conjecture)

The use of GeoGebra software will positively affect the attitudes and beliefs of pre-service math teachers towards math and math education.

Research objectives that allow making operative measures such conjecture was profiled.

Objective 1: Analysis of changes in pre-service teachers' attitudes towards mathematics after the introduction of GeoGebra.
Objective 2: Identification of pre-service teachers' attitude changes towards mathematics teaching after the introduction of GeoGebra
Objective 3: Identification of GG characteristics influencing the transformation of mathematics attitudes of the PSTs

The hypothesis, comprising the core of this study:
You can design, implement and evaluate a teaching sequence, based on the use of GeoGebra, to promote a positive change of PSTs attitudes towards Math and Math Education. GeoGebra usage will develop certain attitudes and competencies at a higher level.

4.2 Participants, Methods and Findings

Participants: The Study participants were 80 PST's in their 2nd to 4th years of study (three courses) pursuing their mathematics teaching degree at the *Sakhnin College for Teacher Education.*

Method:

We developed an Action-Research strategy of investigation.

Research Tools

-Audio and video recordings of the classroom, Interviews, GeoGebra worksheets records, Written protocols of tasks resolution, Suggestions Toolbox and Two adopted questionnaires the "Attitude towards Math" questionnaire and the "I need your opinion" questionnaire. Systematic observations: Attitudes observation file and My personal reflective journal (see in dissertation).

Findings

Attitudes towards Math Questionnaire

I conducted a descriptive SPSS, statistical analysis of the PSTs' answers before carrying out an inferential analysis. Table 4.1 presents the calculations obtained.

Table 4-1 Percentages of PSTs attitudes towards Math according to the opinions reflected in the pre-test and posttest.

	Cognitive		Affective		Applicability	
	Pretest	Posttest	Pretest	Posttest	Pretest	Posttest
Negative	5	7.5	2.5	7.5	7	10
Neutral	10	5	5	7.5	12.5	10
Positive	85	87.5	92.5	85	80.5	80

Table 4.1 shows that the Cognitive component underwent slight improvement, while the Affective component suffered a decline.

Regarding the applicability dimension, the high percentages of the pre-test and post-test participating PSTs appreciated the usefulness of mathematics in real life recognize its potential as a valuable tool in the future.

According to their responses to pre-test and posttest, attitudes towards mathematics before PSTs attitude did not change significantly after the use of GG.

After this first analysis, and since I was interested to know the possible differences significant between the responses of 80 students to the items of the pretest and posttest, were conducted various analyzes of comparison of means. Among them, one test t of student for samples related and two tests not parametric for two samples related types Wilcoxon and signs, all of them with SPSS (Appendix J). These tests revealed differences significant for total students in the affective components (item 12, item 25, and item 29) and in the dimension of applicability (item 18 and item 26).

In what regards the affective component, I found differences in Item 12: Sometimes, I feel tense and uncomfortable in math class. In the pre-test, 22.5 % agree, and in the post-test 35 % agree. Item 25: I would be happy to get my highest marks in math decline from 100 % in pre-test to 95 % in post-test. Item 29: I wish math had never been invented growth from 15 % in pre-test to 25 % post-test.

Continuing with the dimension of applicability, they were found differences in the Item 18: Math will be useful for me in the future. The majority of PST's agree with the item although there is a light decline in the results in the Post-test. Item 26: For my future work, I will need good mathematical skills. Also, in this Item there it's a light decline from 97.5 % agreement in pre-test to 92.5 % in post-test

The inferential analysis performed supports the results of the statistical analysis that pointed to an absence of significant changes between the Pretest and the Posttest.

In summary, this instrument not revealed major changes to the state 's PSTs answers, their attitudes toward the mathematics maintained some stability before and after of the work with GeoGebra.

As after this first analysis no major changes in PSTs attitudes had been observed and given that I wanted to know more of possible significant differences between the PSTs responses to pre-test and post-test items, I decided to perform different means comparison analyses, including a student t-test for related samples and two non-parametric tests for two Wilcoxon-related samples and signs, all with SPSS. After the analysis of Attitude toward Math Education I structured the analysis of attitudes toward the math by components.

Attitude towards Math Teaching

A factorial analysis performed on the questionnaire's items allowed grouping them into four factors that explain 51.131% of the variance.

The items of each factor are presented ordered by their greatest weight in the definition of that factor (an item defines a factor if its weight is greater than 0.40 and higher in that factor than in the other factors). The next step was to interpret the items that define each factor to assign a name to each factor that reflects the information they collect.

Table 4-2. Percentages of PSTs according to opinions reflected in the preliminary analysis of NYO (need your opinion factors).

		Factor 1: Attitude towards the Use of ICT/GG in Mathematics	**Factor 2: Advantages of Working with Mathematics with GG**	**Factor 3: Rejecting Mathematics using GG**	**Factor 4: Work Collaboratively using GG**
Statistics descriptive	Maximum	2.34	2.27	2.25	3.27
	Minimum	-1.73	-1.67	-2.53	-1.21
	Median	0.00	0.00	0.00	0.00
	Typical Deviation	1.00	1.00	1.00	1.00
	Negative	47.5	48.0	37.5	48.0

Attitude (%)	Neutral	0.00	0.00	0.00	0.00
	Positive	52.5	52.00	62.5	52.0

The results of the table above suggest one vision satisfactory of the work done with GeoGebra in math for 60% of PST's, who showed one attitude positive towards the use of ICT in mathematics and felt favorably about the advantages of working the math with ICT and the work collaborative with GeoGebra. Then I approach each aspect separately. Concerning attitudes towards mathematics teaching and learning, the items in this questionnaire allowed to obtain information about the three components considered for these attitudes: cognitive, affective, and behavioral.

In what regards the component cognitive, 72.1% of students claimed to have relied more on their capabilities (item 9), and 65.3% said understanding the math by itself themselves using ICT (item 19). These data suggest that a percentage of the students admitted an improvement in their self-confidence in mathematics (the cognitive component) due to the use of GeoGebra.

Continuing with the component affective, the responses of students about the taste for subject during the work with ICT revealed that 87.5% of them they had enjoyed over the course working with computers (item 2) and only 7.5% (item 21) of the students he said that was not liking the work in mathematics. That is, the use of GeoGebra contributed to an emotional improvement of PSTs towards mathematic learning and teaching. In reference to the behavioral component, the information collected in the survey shows that 85% considered it had participated so more active in mathematics during the work with ICT (item 1).

In summary, the work with ICT had an impact favorably on the improvement of the three components of attitudes toward mathematics teaching and learning for and high percentage of PSTs.

In reference to the attitude of PSTs towards the use of ICT in mathematics , they confirmed that for 63% of them , the use of computers they had helped to feel more confident (item 3) to 85% his use them were motivated to work in mathematics (item 6), the time that contributed to a 70% of PSTs will result more easily the work done in math thanks to the help of GeoGebra (item 10). In all, the students expressed one attitude very positive towards the use of ICT in mathematics.

The attitude of the PSTs towards the collaborative work with GG was very positive, as they themselves were commissioned to reflect their responses to the item that define this factor.

Thus, 77.5% of PSTs preferred to work in this way, while the rest is decanted by the work individually (item 22) and 92.5% of students considered that this methodology of work helped to improve the relationship with peers (item 4).

In summary, the analysis done of this questionnaire suggests one attitude positive towards the use of ICT in mathematics, to the math worked with ICT and to the work collaborative by part of a large percentage of PSTs. In addition , also it has been informed of the benefits that PSTs felt that this methodology of work was on his way to work in mathematics , to the support with their answers that the use of GeoGebra them helped to recognize better their fault and that computers were offered many resources to understand better the math .

Answers regarding mathematical attitudes focused mostly on Perseverance, Critical Spirit, Precision and Rigor, and Creativity.

GeoGebra Features Associated with Development Attitudes towards Mathematics

Thought flexibility: The results obtained indicate that most PSTs experienced an evolution in the attitude: Change your mind based on convincing arguments; some of them also did in the one attitude: Solve the problem in more than one way. Regarding the transformation of the change your mind attitude, the software interactivity and feedback received were associated with that transformation.

Critical spirit: The study revealed that two attributes of GG contributed to the PSTs' improvement of this attitude: the software accuracy in calculation performance and representations. Internalizing the benefits of these attributes led the PSTs to rely on results obtained, knowing that an error occurs when they fail to apply the feature properly (lack of accuracy and/or inadequate strategy). This knowledge encouraged them to check for possible errors and address the matter, unlike strategies used when tasks are solved traditionally.

Perseverance: This attitude is also relevant regarding the motivating function of the software. Thus, we can conclude that the confidence expressed by PSTs in solving

Precision and rigor: The improvement shown by PSTs in this attitude was associated with two GG attributes: precision in calculations and visible representations, which proved to be fast to respond and easy to use. Both features helped PSTs to perform graphic constructions quickly and effortlessly. These useful and relatively easy to use features encouraged many PSTs to use the GG skills acquired and perform all tasks in a rigorous and precise manner, as the program can undo and redo tasks as often as necessary to achieve the proper degree of rigor and precision. In short, the program's accuracy, ease of use and responsiveness, motivated the PSTs to use the technology to solve tasks in a more rigorous way, as it enabled them to undo representations and calculations that were not entirely accurate and consider whether the strategy was adequate or not.

Creativity: Although the analyses presented in the previous section indicated that GG did not contribute to the attitude evolution among most PSTs, it was noted that participants who demonstrated flexible and autonomous thinking, worked more creatively with the software. Improvement in this aspect appeared to have had a positive impact on attitude transformation. That is, the software did not exert a direct improvement regarding this attitude, but it can positively affect it indirectly, by enhancing other attitudes encouraging its development.

Autonomy: Two attributes of GG navigability and constructiveness, are associated with promoting autonomous student behavior when using GG to solve tasks or perform assignments, since the software offers the users many options and opportunities to freely explore (navigability) different strategies, along with constructiveness, enabling to try different modes and routs. The most important benefit of this tool is that it can aid the PSTs, boost their self-confidence, and enable them to be responsible for their own learning. Moreover, the study results indicate that the development of this attitude is associated with several advantages of using this software: it grants the PSTs greater autonomy, allows them to adjust their work rate to their personal situation, and encourages teamwork.

Systematization: Data analysis revealed that regarding this attitude, the most influential attribute of GG was its interactivity, as it facilitated the PSTs' ability to manipulate the structures dynamically and provided them with real-time feedback. This feature enabled them to confirm or reject hypotheses and helped them to discover what geometrical properties were preserved by deforming buildings, and by so doing, helped them synthesize their findings and the way they obtained them. In short, the program provided the users with immediate feedback after each action.

4.3 Discussion Pre-Service teachers' transformations of Math related attitudes

At the beginning of this study and relying on experience, I was confident that working with GG will have a positive effect on the three components; however, the positive effect was more pronounced on the behavioral attitude and less relevant to the cognitive component. It is also worth mentioning that I did not expect a great positive improvement in the affective domain; the fact that it had occurred made the experience highly gratifying.

On the other hand, the PSTs displayed very positive attitudes regarding the GeoGebra experience.

The results of mathematical attitudes analysis presented, were compatible to those obtained by other authors such as Khalil, Khalil and Haq (2019) who investigated the changes evaluated by 24 teachers who assessed their students work with ICT in problem-solving, in relation to participation, ability to analyze a problem and interpreting worksheets: initiative, support application, dedication, defending their ideas and creativity.

This analysis lead to the deduction that the PSTs confidence in GeoGebra usefulness as an appropriate problem-solving tool, improve their involvement and self-confidence, and positively affected their attitudes towards mathematics.

Three attributes of the software were found to positively affect attitudes towards mathematics, and its three major components (cognitive, affective, and behavioral).

For most PSTs, the pleasure and confidence gained in working with GG enabled them to find more interest in the subject matter and become actively involved in solving Math-related tasks, which in turn, increased their confidence in their future success as Math teachers. This positive change in their attitudes towards mathematics was vital for many PSTs and served as activator of its evolution in the cognitive domain.

For PSTs coping with greater cognitive deficits, the positive change in their attitudes towards mathematics due to confidence gained by working with GG, helped them improve some mathematical attitudes, a development that favored improvement of certain abilities. Analysis of personal achievements of given participants indicated that overall, PSTS cognitive improvement has been progressive, due to the positive change in their previous attitudes towards Math.

The data analysis indicates that although working with GeoGebra can be considered as a major factor supporting the observed change in the affective and behavioral attitudes components using GG has moderate to low contribution to the improvement of the cognitive component, as the findings suggest that it had enhanced the performance of tasks of low or moderate difficulty level, but seemed to be of lesser importance in performing of complex tasks.

At almost all sessions, most PSTs exhibited Critical Thinking, Perseverance, Precision and Rigor, Autonomy, and Systematization while working with GeoGebra; they used the software to solve problems assigned in tasks designed by tessellations of the plane (GG tasks). In contrast to the above-mentioned attitudes, rating of change in Flexibility of Thought and Creativity was high only among few PSTs but was not high for most of them.

The main factor affecting change of Mathematical attitude among PSTs was working with GeoGebra. The analysis points out certain attributes and advantages of the program, combined with the growing confidence of the participants in using' GeoGebra in math-related activities, (motivating role of software), contributed to positive development of mathematical attitudes such as Critical Thinking, Perseverance, Precision and Rigor, and Autonomy.

The software assisted several PSTs to adopt systematic approach, although not most of them. It also enhanced Flexibility of Thought and Creativity of several PSTs, but for the latter, perhaps indirectly. As for some of the PSTs whose work with the software did not bring about high-level development in all of their mathematics attitudes, other factors such as peers' interaction and instructor attention were often times found to be rather helpful. Specially for the development of attitudes of Systematization, Flexibility of Thought, and Creativity.

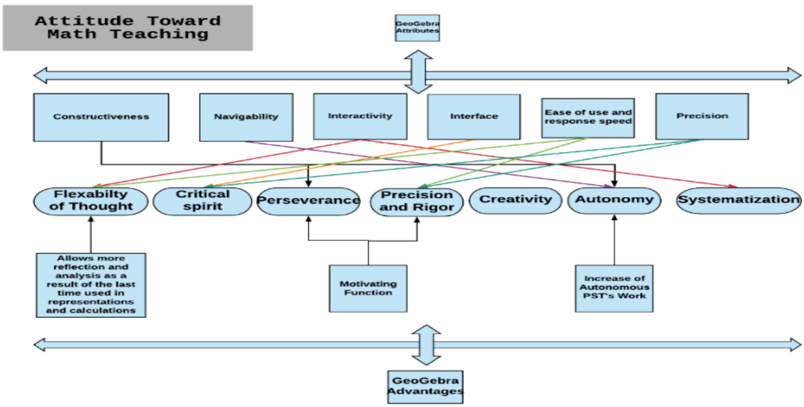
In summary, the results obtained show a greater influence of GeoGebra use upon transformation of attitudes relating to greater involvement in tasks and subject, (Perseverance, Critical Thinking, Precision and Rigor, Autonomy and Systematization) than attitudes of a greater psychological demand (Flexibility of Thought and Creativity).

4.4. Discussion Identification of GeoGebra features affecting attitudes related to Math

The figure below illustrates systematization regarding objective 3: identification of GG characteristics influencing the transformation of mathematics attitudes of the PSTs.

Figures 4.1 summarizes the information presented separately for the results of attitudes final calculation received by triangulation of different factor-related analyses. This process identified the software features associated with attitudinal changes of most PSTs. The information presented may be regarded as a basic scheme reflecting the characteristics associated, with the combined results for each attitude of all the PSTs', with slight variations of each individual student.

Figure 4-2. Causal Model - GeoGebra Features Associated with the Development of Attitudes towards Mathematics Teaching and Learning



4.5. Conclusions

The attitude and the achievements of secondary students to teach and learn with GeoGebra was researched widely (Chan and Leung, 2014).

The Attitude related to GeoGebra use of math pre-service teachers are very positive a priori and after a GeoGebra course (Agyei and Benning,2015)

Two different groups of PSTs are recognized those who did not exhibit notable improvement in any way by working with the software (about 10%), and those who improved their attitudes while working with GG (app. 90%).

The data analysis revealed the importance of collaborative work as being the second influential factor after the software in improving attitudes such as self-confidence, Flexibility of Thought, Creativity and Systematization, and the development of communicative competence.

This analysis also pointed to the collaborative work and interaction among PSTs as being the third important factor, after the software and the pre-service interaction with teacher-instructor, to develop thinking and reasoning and argumentation-proof competencies.

The results show an improvement in affective and behavioral components among most participants but indicated no significant improvement regarding the cognitive component.

Therefore, I can conclude that the experiment of GG introduction has been both rewarding and satisfying. Moreover, the time and effort dedicated to the design and planning of this study was amply rewarded by the PSTs' responses, both on the attitudinal and cognitive levels of the participants, and the professional and personal development experienced by me during the three cycles studies

5. Study 3: Pre-Service Mathematics Teachers’ Ability to Identify Skills Needed to Work with GeoGebra Software

5.1. Research Aims

Conceptual Framework of the Study

According to Özmantar et al. (2010), the components of pedagogical content knowledge, listed below, are examined within the context of the GeoGebra program:

1. The specific skills require to use the program

2. Multiple representations of concepts

3. Identifying and addressing students’ difficulties and misconceptions regarding the new concepts

4. Using built-in methods and strategies to teach concepts

5. Using tools enabling concept assessment-evaluation

Research Questions

1. How do pre-service teachers perceive their teaching ability in combining math content knowledge, pedagogical methods, and technology (TPACK) with the added use of GeoGebra? (questionnaires)
2. Are the pre-service teachers aware of the specific skills required for the efficient use of GeoGebra? (specific skills)

Questions related to their use of GeoGebra to teach mathematical concepts:

3. How, if at all, do the PSTs use GeoGebra to teach Concepts using multiple representations of a concept?
4. How, if at all, do the PSTs use GeoGebra as a means of deriving student difficulties from misconceptions?
5. How, if at all, do the PSTs use GeoGebra’s in relation to methods and strategies for concept instruction?
6. How, if at all, do they do PSTs use GeoGebra to assess/evaluate student learning?
7. How, if at all, do the PSTs integrate the use of GeoGebra in their teaching concepts across all the curriculum?

5.2 Participants, Method and Findings

The research participants were 22 PSTs in years 2-4 of a mathematics education program at Sakhnin College in pursuit of a mathematics teaching degree.

The study focuses on how the PSTs' skills in content, pedagogy, and technology integration were reflected in their projects.

Main Research Tools

The tools are a combination of observable and non-observable tools (Voogt, Fisser, Pareja Roblin, Tondeur & van Braak, 2012).

Observable: Specific Skills observation, GeoGebra Concepts observation

Non-Observable: Self-efficacy Perception Scale for Computer Aided Education Questionnaire, TPACK Questionnaire ,Video Recordings, Interviews, Instructional plans, Self-evaluation form and GeoGebra files.

Findings

The findings will be divided according to the research questions presented above. In each section, the detailed qualitative results of the study are displayed under separate headings within the scope of the TPACK components.

How do pre-service teachers perceive their teaching ability in combining math content knowledge, pedagogical methods, and technology (TPACK)? (Self-Efficacy and Tpack questionnaires)

PSTs' perceptions of their self-efficacy regarding computer-assisted education ($t = .508$, $p > .05$) and their opinions about TPACK ($t = .544$, $p > .05$) were examined. The results did not show any statistically significant differences between the mean scores of pre-test and post-test measures.

The t-test examined whether or not the scores of participants' pre-test and post-test self-reports display a difference regarding their perceived application of techno-pedagogic knowledge in their individual projects. The results of the two questionnaires showed a statistically meaningful difference at a 0.05 significance level ($p< .05$).

The mean scores of the two questionnaires pre-test (= 2.77) and post-test (=3.34) and PSTs' opinions about GeoGebra computer-assisted math teaching had positively improved, although there is no significant statistical difference between their pre-test and post-test opinions

Are the pre-service teachers aware of the specific skills required for the efficient use of GeoGebra? - Specific Skills

Only 16 PSTs integrated the specific skills explicitly stated in the category: "inputs of the function equation." In the "interpret numerical results produced by GeoGebra" category, which requires using all of the specific skills, 20 PSTs did not use them at all.

Nevertheless, significant qualitative gaps were found in the PTSs' projects.

GeoGebra and Teaching Concepts

How, if at all, do the PSTs use GeoGebra to teach Concepts using multiple representations of a concept ?

In the first project, the rate of GeoGebra usage in different representations was found to be low. After examining the peer- and self-evaluations, participants were found, in the second projects, to have increased the specified representations as well as the extent to which GeoGebra was used. The semi-structured interviews with the PSTs concerning the self-evaluations of their first project revealed that most of them did not benefit from GeoGebra's tools for interrelating representations.

How, if at all, do the PSTs use GeoGebra to as a means of deriving student difficulties from misconceptions?

The PSTs' instructional plans of their first projects, including references to objectives, difficulties, misconceptions, errors, and their interrelations, reveal uncertainty. The methods used to address these concepts in practice, and the fact that most participants did not benefit from the computer software when planning their first project are also intriguing.

Review of the PSTs' first and second projects show that most of them are unable to make efficient use of the computer software to overcome student difficulties. Although most PSTs acknowledged the computer software as being convenient to overcome student difficulties,

The fact that most of the PSTs focused on graph difficulties when discussing their second projects suggests that difficulties addressed regarding software type might have been limited.

Another intriguing finding in this context is the fact that the PSTs did not consider the potential problems students might face in terms of GeoGebra use.

How, if at all, do the the PSTs use GeoGebra's in relation to methods and strategies for concept instruction?

In an examination of the methods and strategies preferred by the PSTs in their lessons, the results reveal no significant change in the tools they used between their first and second projects.

PST's uses more than one strategy in their projects.

The most preferred methods and strategies used by PSTs included discovery learning, direct instruction, discussion, questions and answers, and brainstorming in both their first and second projects.

Analysis of the levels of the PSTs' integration of GeoGebra skills in their methods and strategies (as defined in the data analysis section) showed that the number of Level 3 activities increased in the second projects, probably due to the PSTs' exposure to their peers' evaluation and their own self-evaluation of their first project.

How, if at all, do the do PSTs use GeoGebra to assess "student" learning?

The PSTs used different assessment tools for different purposes in their first projects. What drew my attention was that only six pre-service teachers used GeoGebra in the assessment-evaluation process.

Most participating PSTs had integrated technology and formative assessment-evaluations into their second (final) projects.

We can sum up by saying that, in an examination of the assessment-evaluation tools used by the PSTs and their purposes, it appears that the main benefit the PSTs derived from learning to use the technology was expressed by the formative methods they used in their assessment-evaluation processes for the second projects.

How, if at all, do the PSTs integrate the use of GeoGebra in their teaching concepts across all the curriculum

The PSTs' first projects and instructional plans show that most of them tend to prefer goals involving the curriculum statement: "information and communication technologies can be benefitted from." I also found that the PSTs did not present the objectives they had initially set for using GeoGebra for concept teaching and multiple concept representations. Analysis of the goals stated in the instructional plans the PSTs made for the first projects revealed that eight PSTs appear to have mistakenly included activities in their projects above the level of the relevant grade/age-group targeted by the curriculum.

5.3. Discussion and Conclusion

How do pre-service teachers perceive their teaching ability in combining math content knowledge, pedagogical methods, and technology (TPACK)? (Self-Efficacy and Tpack questionnaires)

The Self-Efficacy Scale relating to Computer-Based Education and the TPACK Awareness Questionnaires were applied before and after the integrated course.

Although the quantitative results might give the impression that the PSTs' skills had not been reinforced by the qualitative results, in my opinion, not finding a significant difference between pre-test and post-test was due to the level of awareness and the actual timing in which the testing tools had been applied.

Although the quantitative results may give the impression that no reinforcement of the qualitative results occurred, the reason for finding no significant difference might stem from the fact that the awareness-level pre-test and post-test were conducted after the participating PST's had already become acquainted with TPACK concepts in previous courses.

Performed TPACK vs. self-reported TPACK: The study revealed a mismatch between the performed TPACK and the participants' self-reported TPACK. The self-reported TPACK scores of each participant were higher than their actual observed TPACK behaviors.

One explanation for the mismatch might be potential flaws and differences in the development of the various assessment tools.

Are the pre-service teachers aware of the specific skills required for the efficient use of GeoGebra? (specific skills)

The quality of the first individual projects submitted by the participating PSTs failed to meet my expectations. I was disappointed, for example, by their inability to interpret GeoGebra numerical results. Although they did solve problems requiring this skill, which was highlighted in the discussions, some of them did not use it when they actually should have, as it appears, they did not fully realize its importance. Perhaps some of them did not have an adequate understanding of real numbers. Thus, it seems that PSTs should possess more mathematical content knowledge prior to enrolling in a Teachers College with the explicit intention of becoming Math teachers.

Teaching Concepts with GeoGebra

How, if at all, do the PSTs use GeoGebra to teach Concepts using multiple representations of a concept?

Examination of the PSTs’ instructional plans for the first projects showed that, though many participants used different representations, most of them did not interrelate them, while inter-relations between representations are an important aspect of understanding the subjects conceptually.

An examination of peer- and self-evaluations shows that the participants had devoted enhanced efforts to the issue of interrelating representations in the second project.

Considering this study's results, one could say that the PSTs did not derive much benefit from this component of GeoGebra, especially with regard to interrelating representations.

How, if at all, do the PSTs use GeoGebra to as a means of deriving student difficulties from misconceptions?

An examination of the PSTs' instructional plans for the first projects reveals that they were unable to precisely distinguish the concepts of difficulties, misconceptions, and errors. In addition, the fact that most of the participants did not benefit from the computer software to plan how to address these concepts is intriguing. PSTs’ knowledge of these concepts is important; only with this knowledge will they be able to develop productive and useful strategies for readdressing students’ understandings (Bingölbali and Özmantar, 2009).

The only possible conclusion is that the PSTs should be provided with environments in which they are allowed to select the appropriate software to overcome concept-related difficulties that prevent them from making optimal plans.

How, if at all, do the PSTs use GeoGebra’s in relation to methods and strategies for concept instruction?

The most frequent instructional methods and strategies used by PSTs in practice for both the first and second projects were: discovery, direct instruction, discussion, questions and answers, and brainstorming. Data gathered indicates that the PSTs made progress in technological integration of methods and strategies of specific skills, not unique to GeoGebra software, but using various effective technologies in their projects.

The participants' progress was shown by an increase in the number of Level 3 activities performed in their second projects, affected by inputs received by means of peer- and self-evaluations of the first projects. According to Hughes (2005), achieving transformation and quality learning through technology is possible when technology is used at the third level, alongside the use of appropriate methods and strategies. The fact that the participants started to use technology more efficiently is a significant indicator of progress in this component.

How, if at all, do they do PSTs use GeoGebra to assess/evaluate student learning?

Analysis of data obtained in the context of TPACK's assessment-evaluation component revealed that the PSTs improved their skills in locating relevant information of various technology-assisted tools used for assessment-evaluation and using it in technology-assisted environments for formative and/or summative purposes. The PSTs' instructional plans for their second projects show that their use of multiple assessment-evaluation tools according to their different purposes was quite remarkable.

How, if at all, do the PSTs integrate the use of GeoGebra in their teaching concepts across all the curriculum?

Analysis of the data gathered by qualitative methods indicates that PSTs tend to prefer accomplishments in the curriculum contents involving the statement "information and communication technologies can be benefitted from" to achieve technological integration. The PSTs did not present the objectives set for GeoGebra usage as part of their instruction-plan for the concept they had reflected on, nor for concepts of multiple representations and misconceptions. Thus, generally speaking, the PSTs identified some newly acquired skills while addressing the curriculum-relating attainments. A similar finding was interpreted by Demir (2011).

A review of the study's application of process contents reveals the importance of investigation and discussion of methodology: how attainments on the axis of vertical articulation are taught to a specific grade/age-group. Another point the results shed light on is the lack of materials that can be used in technology-assisted mathematics teaching designed according to the curriculum. By including technology in the process, teaching and learning are fundamentally altered (Kieran and Drijvers, 2006). This leads teachers in the field, as well as the research participants, to use available resources as GeoGebra Applets.

In our case, some PSTs pointed out the lack of activities aimed at practicing the use of GeoGebra to support instruction, either within the program or in the course materials. They claimed that we're unable to plan and create efficient projects because they could/did not use any GeoGebra resource (applets, worksheets, etc.) to assess how their attainments of GeoGebra related to skills, or the lack of them, had been addressed in their instruction presentation

6. General Conclusions and Recommendations

Teachers cannot be expected to begin successful teaching without the use of specialized software for teaching mathematics.

6.1. Why GeoGebra?

GeoGebra it's a software with many characteristics and features. GeoGebra have advantages in manipulation of Geometry that easy presentation and exploration of Geometric Concepts and examples.

A "GeoGebra" course should be integrated into the pre-service teachers' curriculum.

The most applicable alternative to ensure assimilation of GeoGebra program by preservice teachers is the course called "GeoGebra," a course combining introduction of innovative educational technology and pedagogy.

Obviously, in the framework of digital skills required for the 21st century, there is a need to construct an applicable course focusing on specific program designed for a specific age group.

It is recommended to increase the use of mobile phones in the workshop and while preparing tasks (GeoGebra on mobile phones). Mobile devices are a part of the student's daily life, and therefore, their applicable features should be exploited in classrooms as most current students in both schools and universities cannot see the world without mobile technology (Dias & Victor, 2017; Grant, et. al., 2015). However, in my opinion, other techniques like paper and pencil and computer programs should be used when relevant or convenient.

6.2 Recommendation

The Study Instrument: Observation Form developed by me would be use in further research (academic and field level)

Pedagogy: pedagogy courses without the integration of technology and the use of mobile phones are no longer relevant for pre-service teachers.

Regarding a methodology/pedagogy course as part of teachers college curricula – after leading such course in the department training future math teachers, I am convinced that such course cannot be efficient without the integration of digital teaching technologies focused on mathematic investigation.

In my opinion, the introduction of new materials relates more to the availability of competent lecturers rather than to the abilities of the preservice teachers or students' needs.

The emphasis on the usage of cellphones derives of the fact that a vast ratio of the current population, children, students, and teachers alike possess such instruments and use them regularly. Thus, this accessible available resource can be used to connect Math, which is considered a distant, abstract subject to every day's reality.

7. Selected References

A. Y.-J., & Reigeluth, C. (2011). Creating technology-enhanced, learner-centred classrooms: K–12 teachers' beliefs, perceptions, barriers, and support needs. *Journal of Digital Learning in Teacher Education*, 28(2), 54-62.

Bingölbali, E., & Özmantar, M. F., (2009). Mathematical difficulties in elementary schools and solution proposals]. In E. Bingölbali & M. F. Özmantar (Eds.), *Mathematical misconceptions: Reasons and seeking solutions (pp. 1-30)*. Ankara, Turkey: Pegem Akademi.

Crano, W. D., & Prislin, R. (2006). Attitudes and persuasion. *Annual Review of Psychology, 57*, 345-374.

Darling-Hammond, L. (2006). Constructing 21st-century teacher education. *Journal of Teacher Education, 57(3)*, 300-314.

Demir, S., (2011). Two inseparable facets of technology integration programs: Technology and theoretical framework. *Eurasia Journal of Mathematics, Science & Technology Education, 7(2),* 75-88.

Dias, L., & Victor, A. (2017). Teaching and Learning with Mobile Devices in the 21st Century Digital World: *Journal of Multidisciplinary Studies, 2(5),* 340-345.

Dick, T., & Burrill, G. (2009). *Tough to teach/tough to learn: Research basis, framework, and principles for a practical application of TI-Nspire Technology in the Math Nspired series*. Dallas, TX: Education Technology Group - Texas Instruments.

Drijvers, P., Doorman, M., Boon, P., Reed, H., & Gravemeijer, K. (2010). The teacher and the tool: Instrumental orchestrations in the technology-rich mathematics classroom. *Educational Studies in Mathematics, 75(2),* 213-234.

Ertmer, P. A., (2005). Teacher pedagogical beliefs: The final frontier in our quest for technology integration? *Educational Technology Research and Development, 53(4),* 25-39.

Ertmer, P. A., & Ottenbreit-Leftwich, A. T. (2010). Teacher technology change: How knowledge, confidence, beliefs, and culture intersect. *Journal of Research on Technology in Education, 42(3),* 255-284.

Ertmer, P. A., Ottenbreit-Leftwich, A. T., Sadik, O., Sendurur, E., & Sendurur, P. (2012).Teacher beliefs and technology integration practices: A critical relationship. *Computers & Education, 59(2),* 423-435.

Fuchs, K., and Hohenwarter, M. (2005). Combination of dynamic geometry, algebra, and calculus in the software system GeoGebra. *Computer Algebra Systems and Dynamic Geometry Systems in Mathematics Teaching* (pages 128 – 133).

Grant, M., Tamim, S., Brown, D., Sweeney, J., Ferguson, F., & Jones, L. (2015). Teaching and learning with-12 classrooms. *TechTrends: Linking Research and Practice to Improve Learning, 59(4),* 32-45.

Hernández, R. P. & Gómez-Chacón, I. M. (1997). Attitudes in Mathematical Education. Strategies for change. *One Mathematics Didactic Review, 13*, 41-61.

Hohenwarter, M. (2006). Dynamic investigation of functions using GeoGebra. In *Proceedings of Dresden International Symposium on Technology and its Integration into Mathematics Education*. Dresden, Germany. DES-TIME.

Hohenwarter, M., and Lavicza, Z. (2007). Mathematics teacher development with ICT: Towards an International GeoGebra Institute. In Kuchemann, D., editor, *Proceedings of the British Society for Research into Learning Mathematics, volume 27,* pages 49 — 54, University of Northampton, UK. BSRLM.

Hohenwarter, M. (2008). GeoGebra. Dynamic mathematics software. http://www.geogebra.org.

Hughes, J. (2005). The role of teacher knowledge and learning experiences in forming technology-integrated pedagogy. *Journal of Technology and Teacher Education, 13(2),* 277-302.

Jones, C., & Czerniewicz, L. (2010). Describing or debunking? The net generation and digital natives. *Journal of Computer Assisted Learning, 26(5)*, 317-320.

Khalil M, Khalil U, and Haq Z. (2019) Geogebra as a Scaffolding Tool for Exploring Analytic Geometry Structure and Developing Mathematical Thinking of Diverse Achievers. *Int Elect J Math Ed. ;14(2),* 427-434. https://doi.org/10.29333/iejme/5746

Kieran, C. A., & Drijvers, P., (2006). The co-emergence of machine techniques, paper-and-pencil techniques, and theoretical reflection: A study of CAS use in secondary school algebra. *International Journal of Computers for Mathematical Learning, 11*, 300-338.

Koch, A., Heo, M., & Kush, J. C. (2012). Technology integration into pre-service teacher training. *International Journal of Information and Communication Technology Education (IJICTE), 8(1)*, 1-14.

Koehler, M. J., Shin, T. S., & Mishra, P. (2012). How do we measure TPACK? Let me count the ways. In R. N. Ronau, M. L. Niess & C. R. Rakes (Eds.), *Educational technology, teacher knowledge, and classroom impact: a research handbook on frameworks and approaches (pp. 16-31).*

Lei, J. (2009). Digital natives as pre-service teachers: What technology preparation is needed. *Journal of Computing in Teacher Education, 25(3),* 87-97.

Lim, C. P., Teo, Y. H., Wong, P., Khine, M. S., Chai, C. S., & Divaharan, S. (2003). Creating a conducive learning environment for the effective integration of ICT: Classroom management issues. *Journal of Interactive Learning Research, 14(4),* 405-423.

Lyublinskaya, I., & Tournaki, N. (2012). The effects of teacher content authoring on TPACK and on student achievement in algebra: Research on instruction with the TI-Nspire™ handheld. In R. N. Ronau, C. R. Rakes & M. L. Niess (Eds.), *Educational technology, teacher knowledge, and classroom impact: a research handbook on frameworks and approaches (pp. 295-322).* Hershey, PA: Information Science Publishing.

Ma, L., (1999). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States*. Mahwah, NJ: Erlbaum.

Niess, M. L., (2011). Investigating TPACK: Knowledge growth in teaching with technology. *Journal of Educational Computing Research, 44(3),* 299-317.

Niess, M. L., Ronau, R. N., Shafer, K. G., Driskell, S. O., Harper, S. R., Johnston, C.,Kersaint, G. (2009). Mathematics teacher TPACK standards and development model. *Contemporary Issues in Technology and Teacher Education, 9(1),* 4-24.

Nisbet, S. (1991). A new instrument to measure pre-service primary teachers' attitudes to teaching mathematics. *Mathematics Education Research Journal, 3(2)*, 34-56.

O'Cathain A., Murphy E. & Nicholl J. (2010). *Three techniques for integrating data in mixed methods studies* BMJ 2010; 341:c4587

Özmantar, M. F., Akkoç, H., Bingölbali, E., Demir, S., & Ergene, B. (2010). Pre-Service mathematics teachers' use of multiple representations in technology-rich environments. Eurasia *Journal of Mathematics, Science & Technology Education, 6(1),* 19-36.

Pajares, M. F., (1992). Teachers' beliefs and educational research: Cleaning up a messy construct. *Review of Educational Research, 62(3),* 307-332.

Philipp, R. A., Ambrose, R., Lamb, L. L. C., Sowder, J. T., Schappelle, B. P., Sowder, L... Chauvot, J. (2007). Effects of early field experiences on the mathematical content knowledge and beliefs of prospective elementary school teachers: An experimental study. *Journal for Research in Mathematics Education*, 438-476.

Philippou, G. N., & Christou, C. (1998). The effects of a preparatory mathematics program in changing prospective teachers' attitudes towards mathematics. *Educational Studies in Mathematics, 35*, 189-206.

Pierce, R., & Ball, L. (2009). Perceptions that may affect teachers' intention to use technology in secondary mathematics classes. *Educational Studies in Mathematics, 71(3),* 299-317.

Preiner, J. (2008). *Introducing dynamic mathematics software to mathematics teachers: The case of GeoGebra*. Tesis doctoral. Universidad de Salzburgo, Austria.

Prensky, M., (2001). Digital natives, digital immigrants, part 1. *On the Horizon, 9(5)*, 1-6.

Strong, M., (2011). The highly qualified teacher: What is teacher quality, and how do we measure it? New York: Teachers College Press.

Supandi, L., Ariyanto, W., Kusumaningsih, & An, A. (2017). Mobile phone application for mathematics learning. IOP Conf. Series: *Journal of Physics: Conf. Series*. 983(2018), 012106.

Tapscott, D., (2009). *Grown up digital*. New York, NY: McGraw-Hill.

Thomas, K., & Muñoz, M. (2016). Hold the Phone! High School Students' Perceptions of Mobile Phone Integration in the Classroom. *American Secondary Education, 44(3),* 19-37

Tondeur, J., van Braak, J., Sang, G., Voogt, J., Fisser, P., & Ottenbreit-Leftwich, A. (2012). Preparing pre-service teachers to integrate technology in education: A synthesis of qualitative evidence. *Computers & Education, 59(1),* 134-144.

Wachira, P., & Keengwe, J. (2011). Technology integration barriers: Urban school mathematics teachers perspectives. *Journal of Science Education and Technology, 20(1),* 17-25.

Wilkins, J. L. M. (2008). The relationship among elementary teachers' content knowledge, attitudes, beliefs, and practices. *Journal of Mathematics Teacher Education, 11*, 139-164.

Zbiek, R. M., Heid, M. K., Blume, G. W., & Dick, T. P. (2007). Research on technology in mathematics education: A perspective of constructs. In F. Lester (Ed.), *Second handbook of research on mathematics teaching and learning (pp. 1169-1208).*Reston, VA: National Council of Teachers of Mathematics.