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Phd Thesis

Formation of Thinking and Scientific Language in the Primary Classes

ABSTRACT

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While studying mathematics at school, pupils are often required to formulate and test assumptions, to explain and justify conclusions and to prove general theorem or claims. The proof is the mathematical tool through which, by argumentation, the correctness of a mathematical claim is established and given universal validation, or the opposite confirming that the claim is false thus refuting it (Hanna, 1989).

The argument structure represents a variety of ways (Makar, Bakker & Ben-Zvi, 2015). According to the logical structure of the thinking processes, the argument helps the claimant to present his words logically: to express his opinion, to prove it, and sometimes even to end it with a solution proposal. Moreover, a high level of argument expresses a high level of literacy (Glassner& Schwartz, 2001). Students in the elementary schools apply external justification methods, empiric techniques of justification and an analytic justification technique (Flores, 2002).

This work deals with the tendency of elementary school students to prefer using inductive considerations when given arithmetic claims, and when they need to examine reasoning of the inductive kind as a mathematical proof to an arithmetic claim. I will discuss this issue in the literature review, through the students' way of thinking and their knowledge about mathematical proof, and I will focus on students' inductive tendency while dealing with mathematical claims. First, I will present the intuition influence on thinking, then the acceptable deductive and inductive proof methods in mathematics and the empiric science.

The scientist and mathematician, Karl Friedrich Gauss, called mathematics "the Queen of Science" (Wolfershausen& Wolfgang, 1856). Most mathematical theories, like those of physics and biology are based on hypotheses and deduction. Many philosophers believe that mathematics cannot be refuted, and therefore does not fit Karl and Popper definition of science (Popper & Karl, 1995). Nevertheless, in many universities there is a faculty of Natural Sciences and Mathematics, which suggests that the two fields are related but do not overlap.

Another view holds that certain scientific fields (such as theoretical physics) are actually mathematics with axioms designed to fit reality. In any case, mathematics has a lot in common with fields in the exact sciences. The concept of mathematics as the language of physics and other sciences is ancient and is rooted in ancient Greece.

In light of the complexity of the mathematical language, there are stages and limitations in the development of mathematical thinking, so it is very important to know the learner's ways of thinking and cognitive development as a basis for teaching design. Many researchers have referred to the various approaches that deal with the development of early childhood thinking and their implications for mathematical education. The approaches differ in two main dimensions. One is the extent to which the structures and developmental processes are domain-dependent, occurring at a uniform rate in different areas, or occurring at a different pace in different areas, and in the autonomy of the development process-whether it is innate or environment dependent (Dehaene& Cohen, 1995).

Piaget (1965) argues that children are not passive to the environment. On the contrary: they check the environment, seek solutions actively, and of course - ask questions. Piaget focused on the cognitive processes of collecting and processing knowledge (rather than the knowledge that children acquire). He compared the development of intelligence to physiological development.

Piaget's theory (like Freud's and Eriksson's) is a stage theory, characterized by four stages. Piaget has observed four major stages in cognitive development. At each stage, children acquire new intellectual skills based on the previous stages.

It should be pointed out that although the stage sequence is fixed, each individual has his or her own pace. There are also interpersonal and intercultural differences during each stage.

The meaning of concrete operations:

Operation - the action of logical thinking

Concrete - tangible.

According to these terms, I see that students are still influenced by their "new" logical thinking through examples, inductive thinking when they are requested to explain or construe arguments. This can be explained as a stage of logical thinking which relates to the tangible world. If so, this stage is characterized by thinking that reflects reference only to the conceptual world and only concrete examples illustrate the situation. Because thinking is already logical, for the first time, all the characteristics of thought that were lacking in the pre-conceptual phase are acquired. In my opinion, this conceptual purchase differs from student to student. Some students are able to perform logical actions only in situations involving concrete, tangible objects. And there are those who have the ability to understand through symbolic or verbal examples. Such students still need illustrations through examples. In other words, they still have inductive thinking, but the more verbal or symbolic examples are made up of the concrete examples through a graph or simple numerical examples, the better. Such students have more advanced thinking, however, the children at this stage can think of the tangible and not about the possible, the hypothetical.

According to Piaget, the child is gradually released from the grasp of concrete thinking and acquires the ability of abstract thought. Transition from a thinking style is based on the ability to think beyond the tangible. It is therefore possible to utilize the abilities of students who have formal thinking. However, they need concrete or inductive examples to illustrate the situation of a given argument, or before they understand or attack proof that they will have to make inductive inferences

In my opinion, there is a transition stage between the third and fourth stages, or there is room for dividing the two stages (third and fourth) into two parts: third stage, first part, simple concrete operations, and two complex concrete operations. And a fourth, a low deductive first part, and a second deductive part. In other words, I suggest that the student, in order to move from the concrete stage to the formal thinking stage, will be in an intermediate stage that will connect the two stages, which I will call a low deductive stage.

There are three leading approaches to high-order thinking in the field of education: the skills approach, the understanding approach and the tendency approach. High-order thinking is also reflected in the words of Swartz (2008) who proposed the concept of stimulating and invigorating learning in the context of high-order thinking. This learning is based on the constructivist approach of developing a deep understanding of a meaningful subject for the learner. This understanding is achieved through authentic problem solving, during which an emphasis is placed on building the learner's knowledge in a thinking community and developing a self-directed and metacognitive and reflective learner.

Thus, metacognitive and reflective thinking are central to a variety of high-order thinking processes. Reflective thinking, as many scholars following Dewey have defined, is the ability to understand understanding and to think about thinking (Brown, 1987; Flavel, 1979). Metacognition has various roles in promoting high-order thinking processes. Being a supervisor, thinking regulates and controls cognitive metadata processes including complex knowledge of the thought processes as well as cognitive processes and the way they are established. Metacognitive knowledge in learning relates to three parts: knowledge of the learning task, knowledge of thought processes in general and knowledge of self as a person learning and thinking.

Perkins and Ritzhart (2004) who called high-order thinking "good thinking" and presented three essential aspects whose components are sensitivity, inclination and ability.

These three components of mathematical thinking can be linked to this definition in the emotional, language, and cultural aspects; In the tendency aspect, ways of thinking, inductive tendency and intuition; In the ability aspect, the degree of knowledge of mathematical language, mathematically consistent, and ability to represent schematic mathematical representations.

Bloom (1956) explained that focusing on memorizing material, with no attempt to develop more complex thinking skills, makes schooling futile and irrelevant for students. In view of this, the goal is to promote high levels of thinking in learning such as the application of existing knowledge in new cases, and original creation by students based on existing knowledge. In this context, attempts were made to define and rank the key cognitive skills that exist in humans and to rank them according to levels.

Cultivating thinking is a primary goal in all educational institutions and curricula. According to this, every teacher must be aware of how the learner's thinking is being promoted. But this is not a sufficient condition in the classroom situation in a school, "Teacher awareness of the need to advance thinking is a prerequisite for the existence of a challenging and interesting learning process, but the question is whether it is also a Satisfactory condition" (Zohar, 1996, p. 4).

Costa (Costa, 2001), argues that teaching takes place on three levels, through which it highlights the characteristics of the teacher who promotes thinking. The first is teaching for thinking - the teacher creates appropriate thinking conditions, such as: building lesson plans that contain high levels of questioning, integrated lessons with additional areas of interest, co-teaching with teachers in planning and thinking about teaching, and ways of promoting students' thinking in the classroom. The second is the teaching of thinking - direct teaching of thinking skills in different subjects, along with the deliberate and tailored implementation of high-order thinking skills in lessons in different subjects. Third, Teaching on Thinking - High awareness of students 'diversity of learning style and thinking promotes learners' metacognitive processes, explains and demonstrates the epistemological basis of comparison processes, of drawing conclusions and of creative thinking. It is important to note that there is considerable variation among students in relation to intuitive metacognitive development (Veenman et al., 2006).

Many scholars have emphasized the status of arguments in the development of high-order thinking and formal thinking, Kofi (1968) presents the argument as a central concept of Logic Studies (logic theory) and defines it as follows: Each set of arguments that claims to be one of the other claims, which is considered to provide confirmation of this argument ”(P. 27).

Many studies have examined the contribution of arguments to the development of thinking and their variables that influence the level of ability to argue. The findings of these studies are varied and even contradictory. There are findings that reveal natural argumentation skills that humans use, some findings that reveal human limitations in argument construction and assessment, and some that indicate options and directions for dealing with those limitations.

Sometimes a question is asked in class and immediately one student answers, but when this student is asked to reason his answer, he/she gets entangled and fail to explain his/her answer. The ability to give explanation and reasoning in a correct way through logical operations to explanations and proofs is called formal knowledge. On the other hand, the intuitive knowledge according to Fischbein (1987) is a kind of immediate and innate knowledge and there is no doubt concerning its truth. The question here is how can we know something which has not been learned and be sure of something we are not able to explain? The answer is that intuition is not an instinct but rather the outcome of a long learning process but not of the explicit kind which is learned in classrooms at school (Melamed, 2000). Fischbein explained that in the teaching process, after a considerable time which can differ from a person to another, the formal knowledge might turn to be intuitive knowledge even that usually this doesn't happen (Fischbein, 1987). While we can give an interpretation to anything that we have never seen before because giving significant to what is received by our senses is our way for organizing and the intuition in its reality core is a respond to senses, because organized processing of the data is already included within the thinking process and not in the intuition (Sperd, 2000).

Researchers gave different definitions to the term intuition with many meanings. The intuitive thinking is characterized by consistency and immediacy, understood in advanced and gives a sense that there is no need for a proof or examining its correctness (Tirush, Bresh, Tzamir& Klein, 2000). According to Vinner, intuition is a clear perception in which the information is foggy or incomplete, hidden by mechanisms that generate a sense of immediacy, confidence (Vinner, 2000). The intuition imposes itself on the process of raising assumptions, explanations and interpreting facts and also it accepts no alternatives (Tirush, Bresh, Tzamir& Klein, 2000).

In many cases the students rely on the intuition jumping to conclusion and skip the steps built one on the other which are required for proper proof. Sometimes the intuition is visual and iconic images and presentations and therefore hard to phrase it in words (Avinon, 2014). This is the main reason for difficulty of the student who is asked to explain the reasons leading him/her to give their answer for a question in the class. The conclusion or the insight rising from the intuition is direct and needs no mediation by external contents or concepts. According to this, the knowledge is a basic need for proper intuition.

In a situation where a child is given two stacks of coins, he can easily guess which pile has more coins, but it will be difficult for him to estimate which one has the higher value, when each coin has its own different value. The intuition of knowing the larger amount of coins is different from the arithmetic operation of which one has higher value, which is related to the value of each one of the coins. The ability of assessing the higher value is a process of learning and as we grow in age and acquire new knowledge, this process changes from arithmetic operation to intuition that can assess the value of a pile of coins. According to Piaget, organizing a group of sticks from the smallest to the largest is possible only due to the ability of building an image of stairs with them. Choosing the smallest stick and then searching in turn the next in size exists only in age 6.5 to 7 years. Until the child reaches the age 5 to 7 years old, he/she is not able to define concept except through indicating suitable objects or by defining them by the acceptable use of them (Piaget, 1969). The first experiences of the child and the way in which the primary intuitions will flex and turn into thinking are the bases for analyzing similar experiences in the future and will later on become the basis for perceiving the situation in cases of movement questions in mathematics. It is worth noting that mathematics is a wide domain and mathematical intuitions are not necessarily indicating mathematical intuitions in other domains (Sperd, 1994).

Accordingly, the curriculum in Israel (2006) puts an emphasis on solving problems as one of the main subjects in mathematics. The curriculum emphasizes using mathematical tools for the need of solving problems from different contexts of the surrounding and also multi-stage and open-ended problems that require employing previous knowledge and integration between subjects. Similarly, the new curriculum in the United States (Common core, 2010) presents solving problems as the main and first purpose of learning mathematics as well (Hershkovitz, 2014, p. 9).

Cognitive psychologists in the 80s, (Schank& Abelson, 1977; Anderson, 1980) referred to schemes as semantic network that represents relations or scenarios of behaviors, for example, behavior type in a party or restaurant. Howard (1987) describs a scheme as a mental representation of aspects from the world. A scheme has cells related one to another and refer one to another according to the received stimulations, and produce the specific event for the scheme. Rumelhart and Norman (1985) characterize scheme as the data structure for representing the general concept kept in memory. There are schemes for generalizing objects, events, scenes and operations.

Very often the schemes represent prototype of the concepts, and they serve as models to the world. When the data is processed by using schemes we must identify which scheme is the most appropriate for the received stimulation.

Most researchers and educators agree on the significance of the proof in science and mathematics. Nevertheless, there is no agreed definition of the proof in the mathematical education community. The formal definition of the concept of proof, according to Hempel (1945a) is: 'the mathematical proof is deductive-axiom system that starts with basic concepts that are not definable like the point, and axioms that constitute unproved hypothesis within the theory itself. After agreeing about the basic and main concepts, the whole theory was determined, while each concept was defined by basic concepts and axioms, and each theorem was logically obtained from the basic theorems (Hempel, 1945b).

Many researches discussed the issue of teaching mathematics, and mainly the mathematical proof. It is very important to consider teachers' knowledge in the subject and to emphasize it, as well as the restrictions and challenges they face while teaching this subject. Many researches emphasized teachers' mathematical knowledge in teaching the proof.

Many studies have found that there is an inductive tendency among students of different ages. Lovell (1971) and Bell (1976) are among the first to study on this topic. Lovell conducted a study of students aged 14-15, and found that many students at this age had not yet developed the ability to think deductively. In Bell's study, which was also conducted on students aged 14-15, students were asked to prove mathematical claims, and although students knew algebra, most students used numerical examples, which indicated an inductive tendency among students. He argued that students at this age can identify and describe patterns and relationships and to describe them, but they cannot justify them or draw conclusions. In other studies conducted in elementary schools, students tended to use inductive methods to prove mathematical claims. (Aharon, 2000; Healy &Hoyles, 2000; Almedia, 2001; Balacheff, 1988).

Consistency is a central property in mathematics, and a valid mathematical claim while being a necessary logic product of pervious theorems. Mathematical consistency is a significant part in developing mathematical thinking among pupils, while it is very hard to develop mathematical thinking without proper consistency about the relations between the mathematical concepts in all ages. Hereby the main measure for examining the period of mathematical theory: a mathematical theory must be built in a way where a theorem and its negation cannot exist in that theory simultaneously (Tirush, 1995, p. 329).

Mathematicians used to consider the mathematic domain as a consistent domain composed of mathematical axioms and theorems that are not prone to be contradicting and referred to these theorems as definite fact. They referred to it as representing the world of nature and it is a consistent content. This view and perception were valid until the 19th century, but after the non-Euclidean geometries in the 19th century, different reference took place to this content domain. According to this new perception the mathematical theory is perceived as valid as long as the internal consistency is kept in it, and a mathematical claim is valid if she it is derived, through logical conclusions, from a given system of basic and axiom concepts (Hempel, 1945a).

This research is based on three studies, held in three Arabic sector schools in Israel. **Study 1** is quantitive study, in which 267 pupils in grades 4 to 6 in elementary schools in the Arab society participated. **Study 2** is a qualitative study which included 12 teachers of the pupils in study 1. **Study 3** contains further statistical analysis designed to test the validity of a scheme that describes the evolution of thinking.

As I have shown in the schema describing the development of formal thinking in mathematics and in general, there is a process of recurring feedbacks throughout the school years and parallel to the student's psycho-intellectual development stages. This scheme was validated through a thorough examination of the variables using quantitative and qualitative techniques. Its explanatory ability for the phenomenon described is very high, in keeping with the research literature that preceded it and offering solutions to material problems arising from previous writing.

The findings of the present study indicate that the process is not as linear and unidirectional as Piaget's theory might suggest. I think this is a claim that coincides with the personal experience of teachers who teach math and other scientific subjects and is intuitively understandable to them. Therefore, I believe that the descriptive framework proposed in this study may help to somewhat alleviate the contradiction that a large number of teachers experience between their formal knowledge of students' intellectual development and what they see in practice.

Therefore, a significant reform is called for. Unfortunately, in Israel, reforms in mathematics education have concentrated on the 11th and 12th grades in recent years, mainly because in these grades the results can be seen in the matriculation certificate grades. However, the findings of the present study place the emphasis for those who want to make a real change in the quality of mathematical education in order to reap the fruits in the coming years, on elementary schools and school teachers.

ABSTRACT – Practical studys

Purpose: This study examines the preferences of elementary school students in using inductive considerations when given arithmetic claims, and when they review inductive reasoning as a mathematical proof to an arithmetic claim, as well as teachers' attitudes toward types of reasoning.

Methodology: A survey in which participated 267 pupils from the Arabic sector in three different elementary schools in Israel, in grades 4 to 6. The survey, based on the math reasoning tasks by Healy and Hoils (1998), is comprised of Algebra and Geometry reasoning tasks. Alongside the task, a semi-constructed interview was administered to 12 math teachers in these schools.

Results: The study findings support the research hypotheses that (a) There will be a difference in the students' preferences towards the types of thinking, between grades 4 and 6; (b) Sixth graders will be less likely to accept tautologic and inductive reasoning than fourth graders; (c) Elementary school pupils tend to prefer empirical arguments (such as inductive and example) as their approach rather than the arguments that they believe will receive the highest scores from their teachers. However, findings do not support the hypothesis that there will be a difference in teachers' preferences towards different types of thinking. The research findings and their practical implications are discussed together with recommendations for teachers and educators in the field of mathematics and teacher training.

Selected Bibliography

1. Avinon, Y. (2014). The cultivation of intuitive thinking in and around education, 6. Kibbutzim College.
2. Bloom, B.S. (ed.) (1956). Taxonomy of educational objectives: The classification of educational goals: Handbook I, cognitive domain, Longmans, Green, Toronto.
3. Brown, A. (1987). Metacognition, Executive, Control, Self Resolution and other more Mysterious Mechanisms. In F.E. Weinet& R.H. Kluer (Eds), Metacognition. Motivation, and Understanding. Hillsdale, Erlabaum. NY.
4. Common core State Standards for Mathematics (2010). [retrieved September, 2013, from:] http://www.corestandards.org/assets/CCSSI–Math%20Standards.pdf .
5. Dehaene, S. & Cohen, L. (1995). Toward an anatomical and Functional Model of Number processing. Mathematical cognition, 1(1), 83-120.
6. Fischbein, E. (1987). Intuition in science and mathematics: An educational approach. Dordrecht, Holland: D. Reidel.
7. Flavel, J. H. (1979). Metacognition and Cognitive Monitoring – A New Era of Cognitive-Developmental Inquiry. American Psychologist, 34, 906-911.
8. Flores, A. (2002). How Do Children Know That What They Learn in Mathematics is True?, Teaching Children Mathematics, 8, pp. 5, 269-274.
9. Glasner, A. & Schwartz, B. (2001). From the Benefactor called for the understanding of the Naqara: Lailat in the texts. Script, 2, pp. 43-11.

10. Hanna, G (1989). Proofs that prove and proofs that explain In G Vergnaud, J. Rogalski, & M. Artigue (Eds.), Proceedings of the International Group for the Psychology of Mathematics Education Vol II, (pp. 45-51), Paris.
11. Harel, G. (1998). Two dual assertions: The first on learning and the second on teaching (or vice versa). American Mathematical Monthly, 105, pp. 497–507.
12. Hempel, C. (1945a). On the nature of mathematical truth. American Mathematical Monthly 52, pp. 543-556.
13. Hempel, C. (1945b). Geometry and empirical science. American Mathematical Monthly 52, pp. 7-17.
14. Hershkowitz, S. (2014). Is this a problem?: Numerical Problems in Natural Numbers in Elementary School. CET: The Center for Educational Technology - Tel Aviv.
15. Makar, K., Bakker, A., & Ben-Zvi, D. (2015). Scaffolding norms of argumentation-based inquiry in a primary mathematics classroom. ZDM Mathematics Education, 47, pp. 1107 – 1120.
16. Melamed, A. (2000). Intuition in changing worlds. Ala, 26, pp. 9-12. New York: Macmillan.
17. Perkins, D., &Ritchhart R. (2004). When Is Good Thinking? In D. Y Dai., R.J. Sterenberg (eds.), Motivation Emotion and Cognition. Integrative Perspective on Intellectual Functioning and Development. Lawrence Erlbaum. N.J. USA.
18. Piaget, J. (1965). The Child's conception of Number. New York: W.W. Norton & co.
19. Scheuermann, A. M., & van Garderen, D. (2008). Analyzing students' use of graphic representations: Determining misconceptions and error patterns for instruction. Mathematics Teaching in the Middle School. 13(8), pp. 471-477.

20. Schoenfeld, A. (1985). Mathematical problem solving. Orlando, FL: Academic Press.
21. Sigel, I.E. (1991).Parents Influence on Their Childrenws Thinking. In A.L. Costa (Ed), Developing Minds. A Resource Book for Teaching Thinking (pp. 43–46). ASCD Pablication USA.
22. Sperd, A. (1994). On mathematical thinking, mathematical research and curriculum development - sensing with Prof. Shimshon A. Amitzur. Ala, 15, pp. 7-12.
23. Sperd, A. (2000). Intuition as a metaphor. Ala, 26, pp. 42-49.
24. Stacey, K., & Vincent, J. (2009). Modes of reasoning in explanations in Australian eighth-grade mathematics textbooks. Educational Studies in Mathematics, 72, pp. 271–288.
25. Stanovich, K. (1999). Who is rational? Studies of individual differences in reasoning. Mahwah. N.J.: Erlbaum.
26. Stavy , R. & Tirosh , D. (2000) . How Students (MIS) understand science and mathematics, Intuitive rules. Teachers College Press.
27. Stein, N. L., & Miller C. A. (1993). The development of memory and reasoning skill in argumentative contexts: evaluating, explaining, and generating evidence.
28. Stylianides, A. J. (2007a). Introducing young children to the role of assumptions in proving. Mathematical Thinking and Learning, 9(4), pp. 361–385.
29. Swartz, R. J. (2008). Energining Learning. Educational leadership, 65 (5), 26-31.
30. Tall, D. (1989). The nature of mathematical proof. Mathematics Teaching, 128, pp. 28-32.
31. Tall, D. (1990). Inconsistencies in the Learning of Calculus and Analysis. Focus on Learning Problems in Mathematics, 12, pp. 49-63.

32. Tavares, M. L., Jimenez-Aleixandre, M. P., & Mortimer, F. E. (2010). Articulation of Conceptual Knowledge and Argumentation Practices by High School Students in Evolution Problems. Science and Education, 19, pp. 573-598.
33. Tirosh, D. Barash, E., Tzamir, P. and Klein, R. (2000) Psychological aspects in teaching mathematics. Tel Aviv: The Mofet Institute.
34. Tsamir, P. (2005). Enhancing prospective teachers' knowledge of learners' intuitive conceptions: the case of "Same A - Same B". Journal of Mathematics Teacher Education. 8, pp. 469-497.
35. Tsamir, P., Tirosh, D., &, Levenson, E. (2008). Intuitive non-examples: The case of triangles. Educational Studies in Mathematics. 69, pp. 81-95.
36. Van Dormolen, J. (1977). Learning to understand what giving a proof really means. Educational Studies in Mathematics, 8, pp. 27-34.
37. Veenman, M. V. J., &Beishuizen, J. J. (2004). Intellectual and metacognitive skills of novices while studying texts under conditions of text difficulty and time constraint. Learning and Instruction, 14, 619-638.
38. Vinner, S. (2000). Satisfaction of the need for certainty - intuition. Ala, 26, pp. 17-22.
39. Waltershausen, Wolfgang Sartorius von (1856). Gauss zumGedächtniss (repr. 1965). Sändig Reprint Verlag H. R. Wohlwend. ASIN B0000BN5SQ. ISBN 3-253-01702-8.
40. Watson, A. & Mason, J. (2005). Mathematics as a constructive activity: Learners generating examples. Mahwah, NJ: Lawrence Erlbaum.
41. Zohar, A. (1996). Learn, think and learn to think. Jerusalem: Branco Weiss Institute for Cultivation of thinking.